NEROGRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

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Surface loading in view of the Earth’s deformability

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Overview

1. Process of surface loading

   concept of surface loading – observables - Earth system processes

2. Modelling of surface deformation

   load Love number, Green's functions, Stoke's coefficients, assumptions behind

3. Determination of load Love numbers

   Continuum mechanical field equations – earth model

4. Beyond elasticity

   Anelasticity to viscous flow

5. Glacial isostatic adjustment

   linear-trend signal – contributes to gravity signal due to ongoing adjustment of the earth

Concept of surface loading

Deformational response of the earth to a surface mass ...

Surface mass density at earth surface, a, is defined as

$$\sigma(a,\Omega) := \int_{r_0} \rho_\sigma(r,\Omega)\,dr\ .$$

$\Omega = (\theta, \varphi)$ is the coordinate pair,

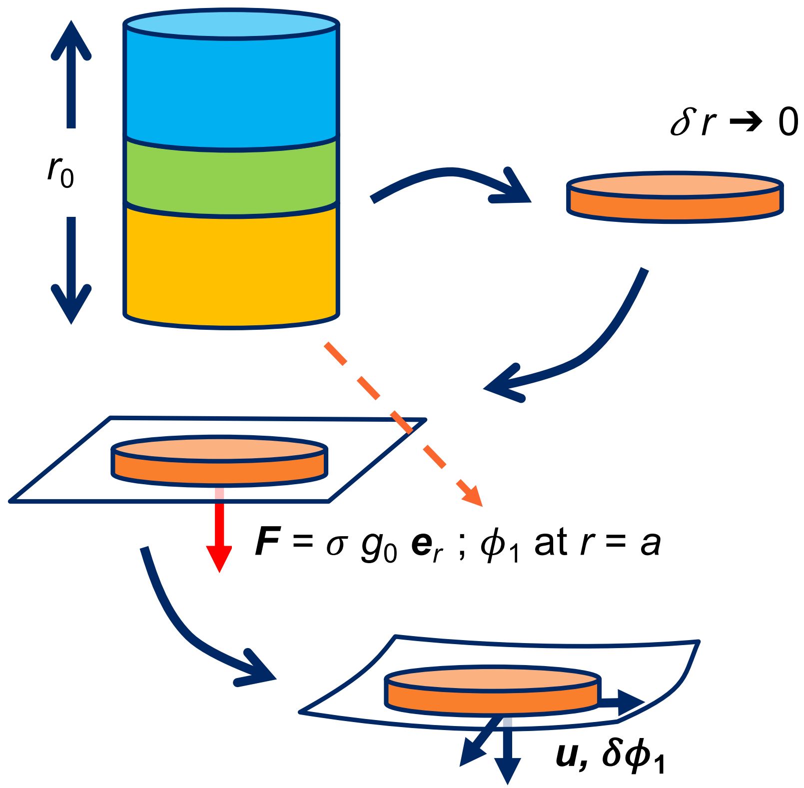
r_0 is the considered radial range,

ρ_σ is the radial density distribution of the load.

... acting as a gravitating load.

(Rem: ϕ_1 may be also calculated from ρ_σ(r))

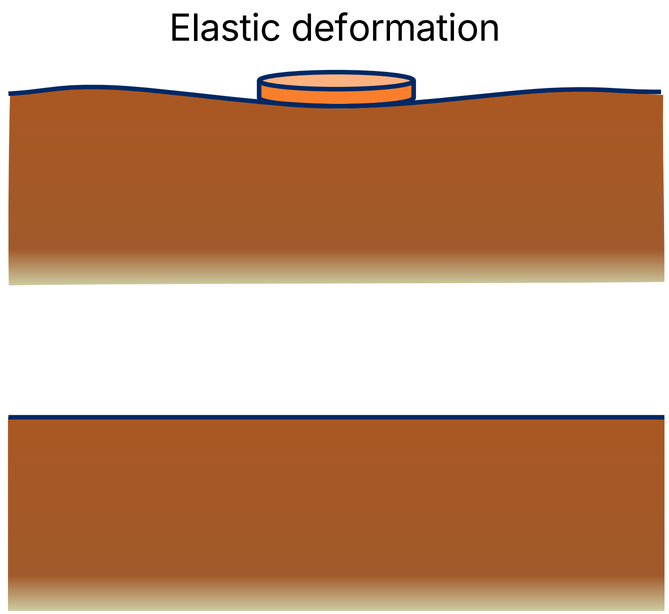
... resulting in a displacement and change of the gravity field.



Geodetic observables

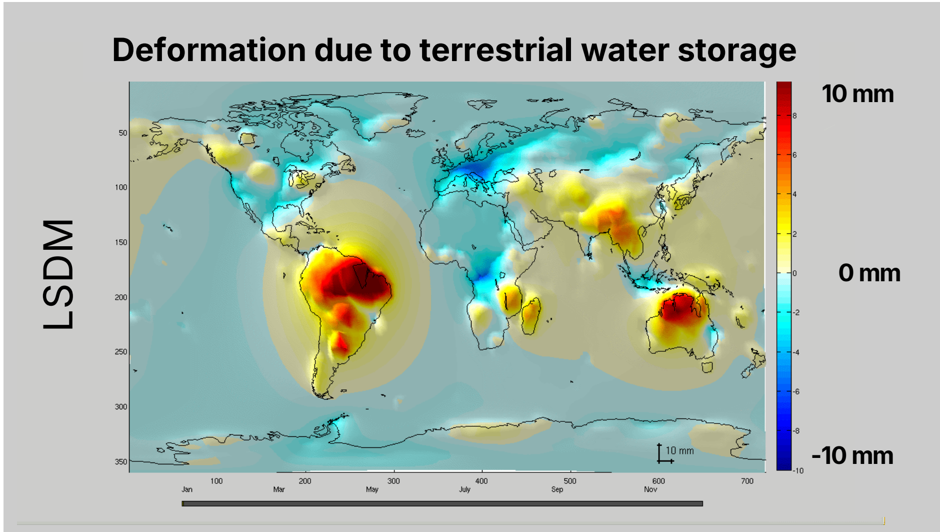
Time variability of loading is the main signal of interest

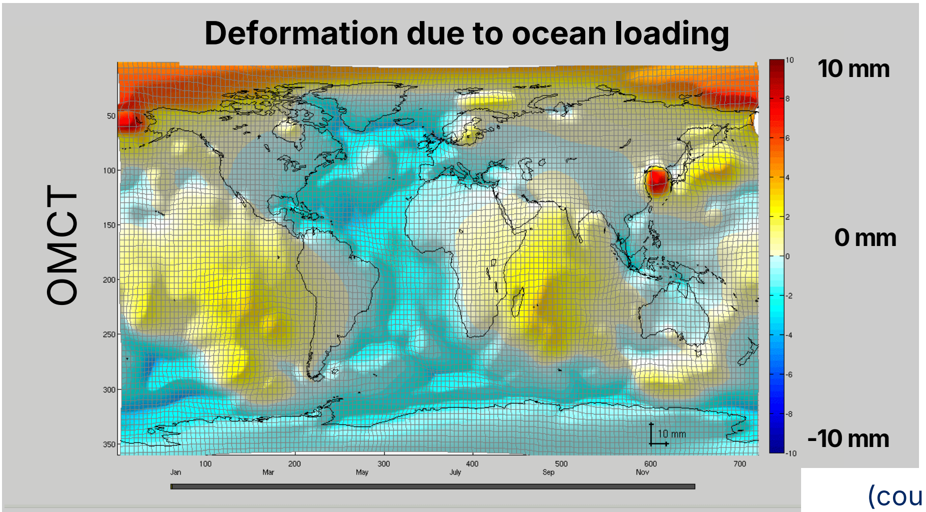
- Surface displacement
  - Leveling, triangulation (optical methods)
  - Global Navigation Satellite Systems (GNSS)
  - Very long Baseline Interferometry (VLBI)
  - Satellite altimetry (SLR)
  - Interferometric Synthetic Aperture Radar (InSAR)
- Gravity
  - Surface gravity (AG)
  - Satellite gravimetry (Sputnik → Champ, **GRACE(-FO)**, **GOCE** → NGGM, GRACE-C, ...)
- Relative sea level
  - Tide gauges

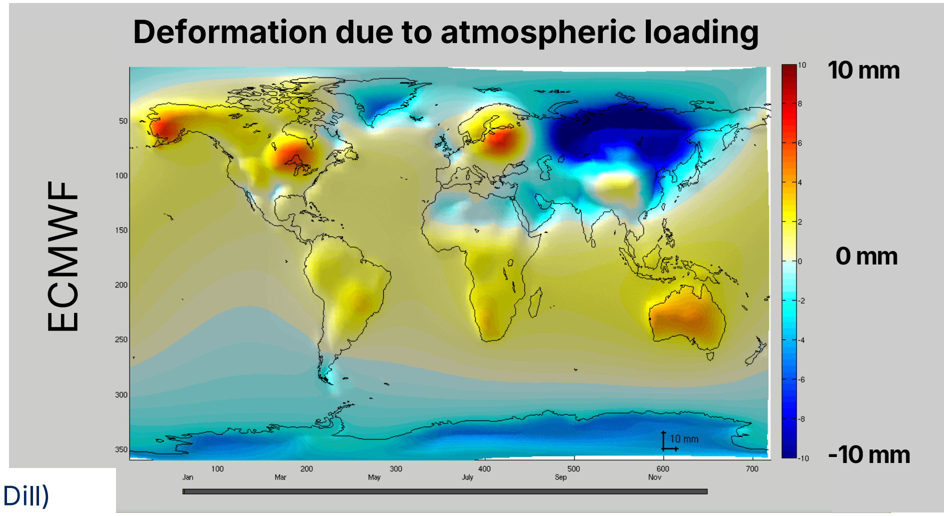


Loading processes (sub-annual)

- Terrestrial water storage
  - Land surface modelling
- Ocean loading (non-tidal)
  - Ocean general circulation modelling
- Atmosphere
  - Weather system modelling

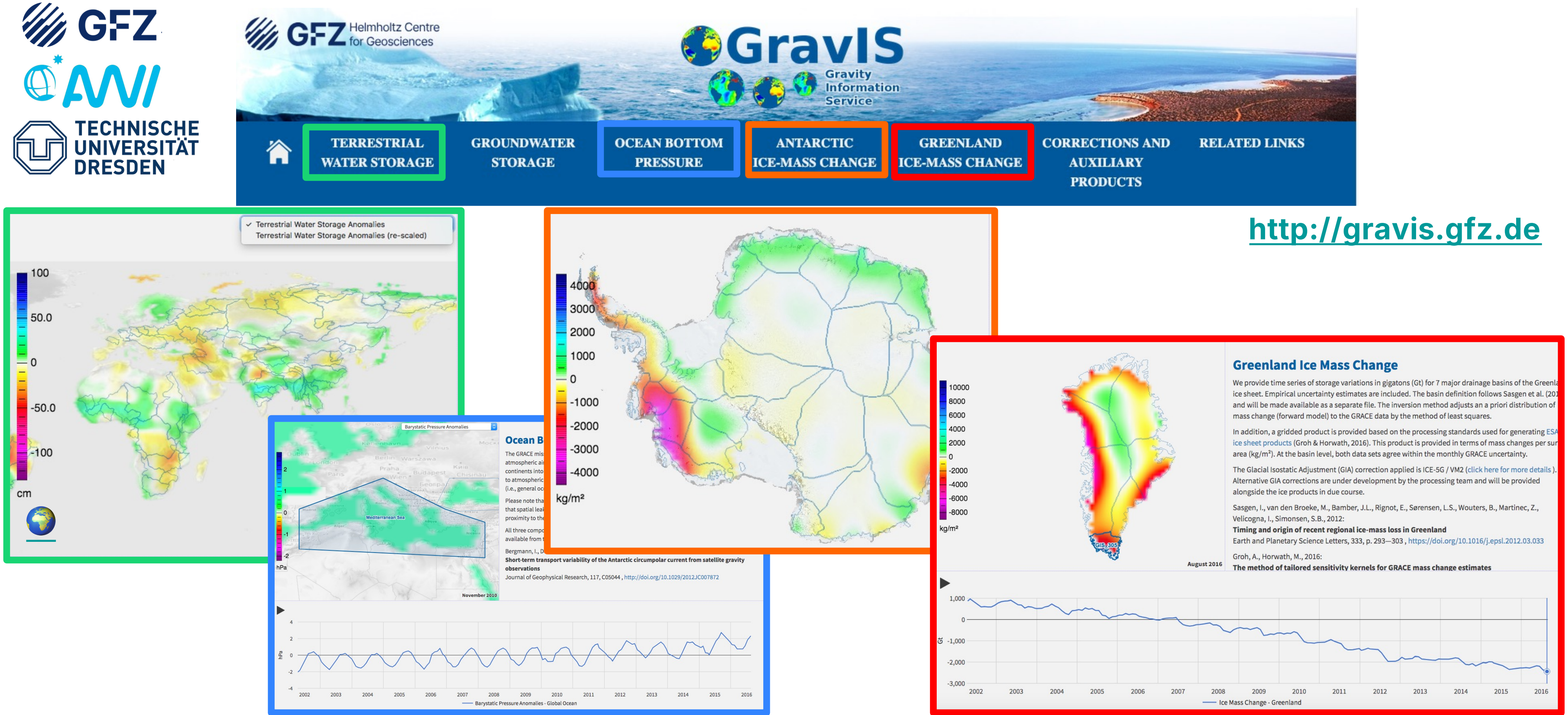






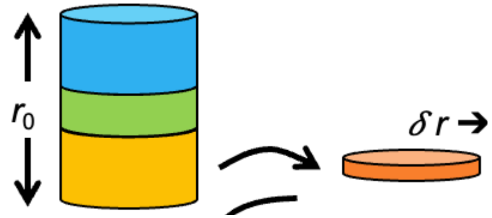
(courtesy Robert Dill)

GRACE/GRACE-FO Mass Anomalies: GravIS

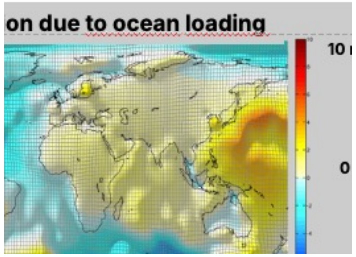


http://gravis.gfz.de

Resumé 1



- Surface mass is considered as a surface mass density applied at the earth surface
- Seasonal loading processes of surface fluids are at the order of 1 cm



- Earth system modelling provides spatial and temporal load distribution
- GravIS is an online tool and can be used as basis for surface mass distribution constrained by satellite missions like GRACE(-FO)

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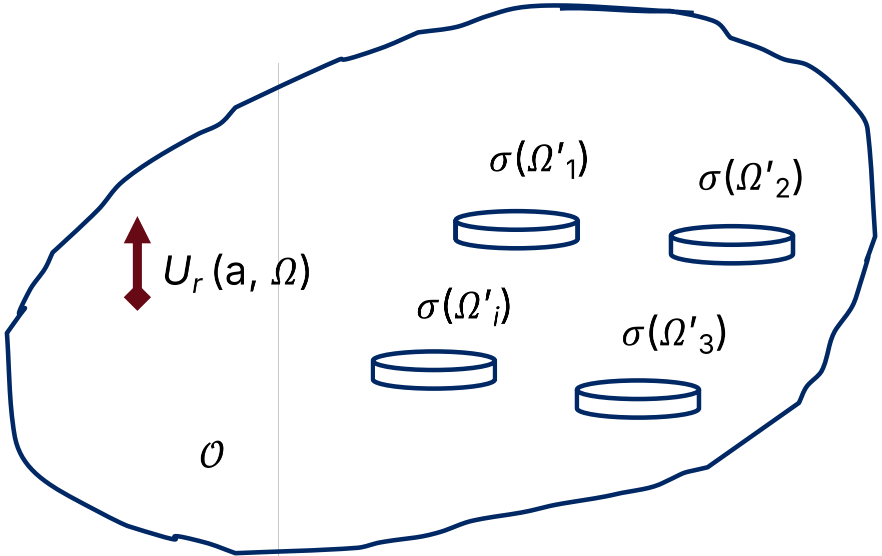
Surface load modelling

$$u_r(a,\Omega) = \sum_i g_u(\Omega,\Omega'_i)\,\sigma(\Omega'_i) \quad \text{with } \Omega'_i \in \mathcal{O}$$
$$= \int_{\mathcal{O}} g_u(\Omega,\Omega')\,\sigma(\Omega')\,d\Omega'$$
$$= g_u(\Omega,\Omega') \star \sigma(\Omega')$$



Spherical symmetry of Earth structure means

$$\Rightarrow \quad u_r(a,\Omega) = a^2 \int_{\mathcal{O}} g_u(a,\gamma)\,\sigma(\Omega')\,d\Omega'\,, \quad \gamma = |\Omega - \Omega'|,$$

with Green's function,

$$g_u(a,\gamma) = \sum_l G^u_l(a)\,P_l(\cos\gamma) \quad \text{and Legendre polynomial } P_l(\cos\gamma),$$

Green’s functions – Load Love numbers

Displacement of reference potential and surface:

$$g_e(\gamma) := a/M_e \sum_l (1 + k_l)\, P_l(\cos\gamma)\ ,$$
$$g_u(\gamma) := a/M_e \sum_l h_l\, P_l(\cos\gamma)$$

Farrell 1972. Deformation of the Earth by Surface Loads. Rev. Geophys. 10, 761.

where h_l and k_l are the load Love numbers.

$$e(\Omega) = \frac{3}{\bar{\rho}} \sum \frac{1 + k_l}{2\,l + 1} \Sigma_{lm}\, Y_{lm}(\Omega)$$
$$u(\Omega) = \frac{3}{\bar{\rho}} \sum \frac{h_l}{2\,l + 1} \Sigma_{lm}\, Y_{lm}(\Omega)$$

Convolution

1. Multiplication

$$e(\Omega) = \frac{3}{\bar{\rho}} \sum \frac{1+k_l}{2\,l+1} \Sigma_{lm} Y_{lm}(\Omega)$$

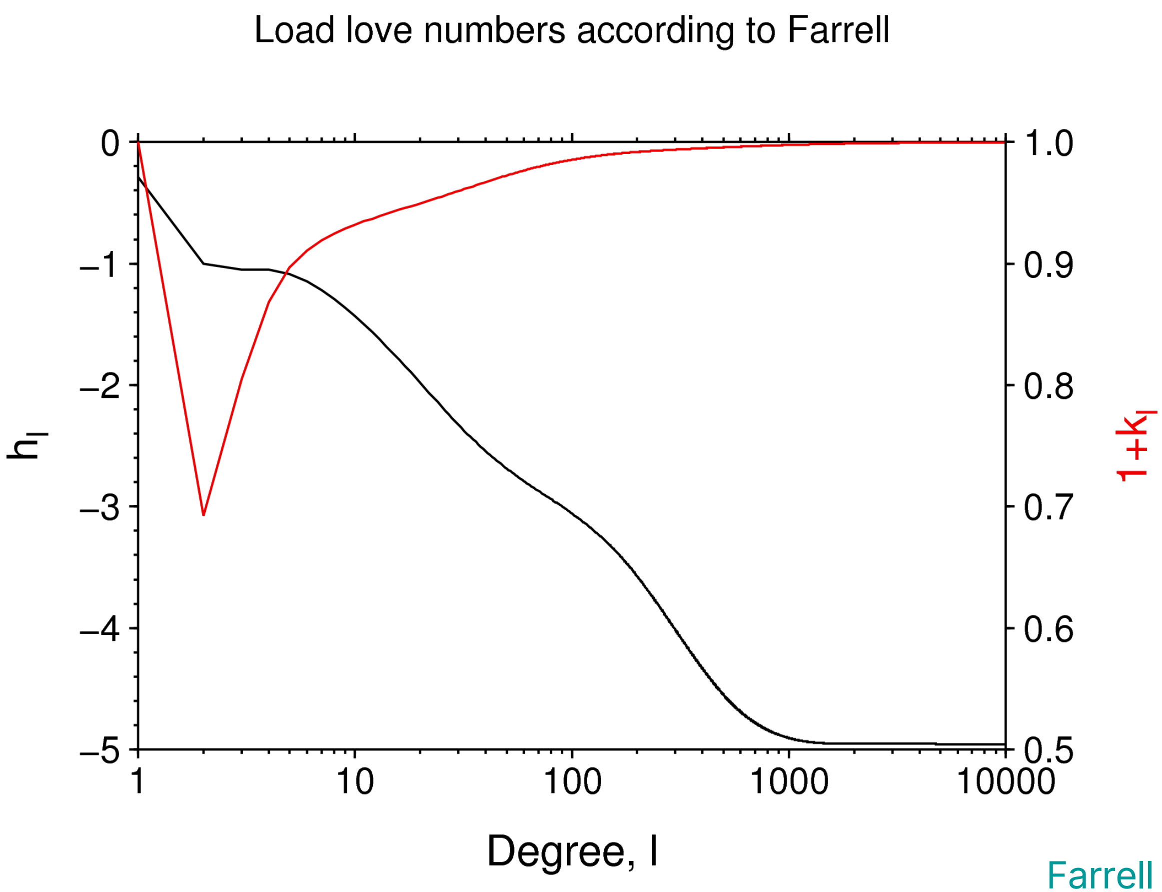
   a) Transformation of load into spectral domain
   b) Multiplication with load Love numbers
   c) Transformation back into spatial domain

2. Convolution

$$e(\Omega) = a^2 \int_{\mathcal{O}} g_e(\gamma)\,\sigma(\Omega')\,d\Omega' , \quad \gamma = |\Omega - \Omega'|$$

   a) Solution of convolution directly in the spatial domain

Load Love numbers



- Quasi continuous for $l > 1$
- $l = 0$:
  - is not excited if mass is conserved
- $l = 1$: depends on ref. system
  - $1 + k_1 = 1$: centre of solid earth
  - $1 + k_1 = 0$: centre of mass
- for $l \to \infty$:
  - $h_l \to$ const.
  - $k_l < 0 \to 0$
    - *reduced compensation*

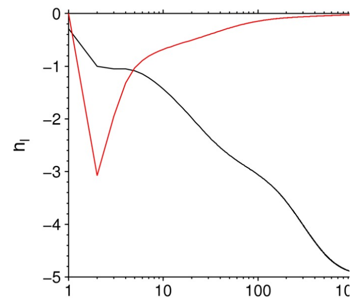
Farrell 1972. Deformation of the Earth by Surface Loads. Rev. Geophys. 10, 761.

Characteristics of Love numbers

- Response of solid earth
  - spherical symmetric (average crust, no difference between continent and ocean)
  - **elastic**
  - Earth is considered as gravitating body
- h, k, l describe vertical, potential and horizontal displacement.
- Two separate processes
  - load Love numbers (response to surface pressure)
  - tidal love numbers (response to tidal forcing)
- They are valid for instantaneous processes

Resumé 2

$$g_u(a,\gamma) = \sum_l G_l^u(a)\, P_l(\cos$$



- LLNs or Green’s functions describe loading, i.e., deformation as well as gravity change due to surface masses.
- Lateral variability is not considered in LLNs.
- Degree 0 is not excited if mass is conserved.
- LLNs of degree 1 depend on considered reference frame and describe geocenter motion.

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Determination of load Love numbers

- Solve the field equations of a gravitating elastic continuum
- Representation in Lagrange coordinates
  - Material coordinate system moves with the deformation,
    i.e., the surface mass as the observer automatically move with the surface displacement
  - The potential equation is solved in local coordinates,
    i.e., the change of the gravity potential changes according to the reference state.

Ingredients

- Continuum mechanical formulation

  motion of a solid -> topology of a continuum
- Coordinate system

  knowledge of state at displaced material point -> Langrange' formulation
- Constitutive equation

  stress state depends on deformation state -> Hooke's law
- Potential equation

  mass redistribution changes gravitational field -> Poisson equation

Field equations

Lagrange' formulation of Cauchy equation of motion – Potential equation – Constitutive equation

$$\nabla \cdot \boldsymbol{t} - \rho_0 \nabla \phi_1 + \nabla \cdot (\rho_0 \boldsymbol{u})\, \phi_0 - \nabla(\rho_0 \boldsymbol{u} \cdot \nabla \phi_0) = 0 \quad \text{momentum equation}$$

$$\nabla^2 \phi_1 + 4\pi G \nabla \cdot (\rho_0 \boldsymbol{u}) = 0 \quad \text{Poisson equation}$$

$$\boldsymbol{t} = \lambda \nabla \cdot \boldsymbol{u} + \mu (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T) \quad \text{Hooke's law}$$

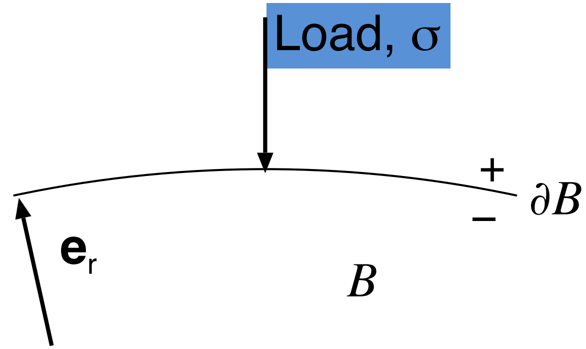
inside the earth, $\mathcal{B}$: displacement, $\boldsymbol{u}$ – stress, $\boldsymbol{t}$ – potential, ϕ_1.

Material parameters: density, ρ_0 – Lamé's first parameter, λ – shear modulus, μ

Rem.: In Eq. 1 inertial forces are neglected on the right

Boundary conditions

Loading of surface mass – free slip – continuity of gravitational potential – potential of surface mass



$$
\begin{aligned}
\boldsymbol{e}_r \cdot \boldsymbol{t}^- \cdot \boldsymbol{e}_r &= -\boldsymbol{e}_r \cdot \nabla \phi_0(a)\, \sigma \\
\boldsymbol{t}^- \cdot \boldsymbol{e}_r - (\boldsymbol{e}_r \cdot \boldsymbol{t}^- \cdot \boldsymbol{e}_r)\, \boldsymbol{e}_r &= 0 \\
[\phi_1]_-^+ &= 0 \\
[\nabla \phi_1]_-^+ \cdot \boldsymbol{e}_r + 4\,\pi\, G\, \rho^- \,(\boldsymbol{u}^- \cdot \boldsymbol{e}_r) &= 4\,\pi\, G\, \sigma
\end{aligned}
$$

Spectral representation

Displacement

$$\boldsymbol{u}(r,\Omega) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l}\left[U_{lm}(r)\,\boldsymbol{S}_{lm}^{(-1)}(\Omega) + V_{lm}(r)\,\boldsymbol{S}_{lm}^{(+1)}(\Omega) + W_{lm}(r)\,\boldsymbol{S}_{lm}^{(0)}(\Omega)\right]$$

with $\Omega = (\theta,\ \phi)$. Potential perturbation

$$\phi_1(r,\Omega) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l}\Phi_{lm}(r)\,Y_{lm}(\Omega)\ ,\ e = \frac{\phi_1}{g_0}$$

Y_{lm} skalar spherical harmonics,
$\boldsymbol{S}_{lm}^{(\pm 1)}$ vector spherical harmonics of spheroidal field,
$\boldsymbol{S}_{lm}^{(0)}$ vector spherical harmonics of toroidal field,
$U_{lm}, V_{lm}, W_{lm}, \Phi_{lm}$ radial functions of coefficients.

Martinec, Z. 2000. Spectral—finite element approach for three-dimensional viscoelastic relaxation in a spherical earth, GJI, 142, 117—141.

Excursion: Degrees 0 and 1

$$\int_\Omega Y_{lm}\, d\Omega = \sqrt{4\pi}\,\delta_{l0}\,\delta_{m0}\ ,$$

$$\int_\Omega \sigma\, d\Omega = 0 \quad \Rightarrow \quad [U, E]_{00} = 0$$

Conservation of mass

$$\int_\Omega \boldsymbol{S}^{(\lambda)}_{jm}\, d\Omega = \sqrt{\frac{4\pi}{3}}\,\delta_{j1}\,(2\,\delta_{\lambda 1} + \delta_{\lambda\,-1})\,\boldsymbol{e}_m\ ,\ \text{with}\ \lambda = \pm 1,\ 0$$

$$\boldsymbol{u}_{\mathrm{CF}} := \frac{1}{A}\int_{\partial V} \boldsymbol{u}\, dS \propto [U, V]_{1m}\Sigma_{1m}$$

$h_1 + 2\, l_1$

$$\boldsymbol{u}_{\mathrm{CM}} := \frac{1}{M}\left(\int_V \rho\, \boldsymbol{u}\, dV + \int_{\partial V} \sigma \boldsymbol{r}\, dS\right) \propto \boldsymbol{\Phi}_{1m}\,\Sigma_{1m}$$

$1 + k_1$

Solution

- Separable partial differential equation in ϑ and *r*
  - *transformation into spectral domain*
  - dependency on co-latitude is solved by Legendre polynomials
  - *dependency on radius is solved by standard method (e.g. Runge-Kutta)*
- Parametrisation of ρ, λ, μ by given earth model
- Forcing is point load at $\vartheta = 0$
- Solutions are load Love numbers
- Summation to get Green's functions

Elastic material parameters

Radial structure of density, and two elastic moduli

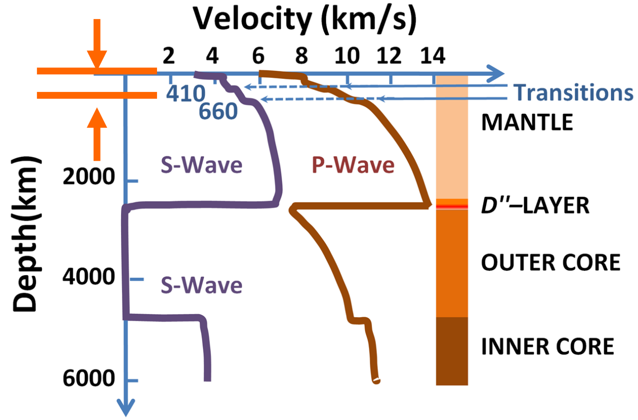
- Seismic velocities
  - Inversion of wave propagation
  - Results in two parameters:

$$V_P = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

$$V_S = \sqrt{\frac{\mu}{\rho}}$$

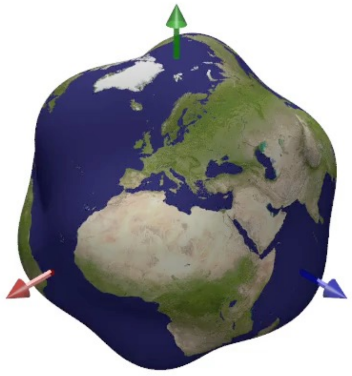
- Third parameter by inversion of free oscillations
  - Free oscillations are excited by large earthquakes
  - Characteristic frequencies
  - Inversion for all three parameters as function of radius

(Rem.: Free oscillations are solutions of same set of field equations, but with the inertia term considered)



By Brews ohare - Own work, CC BY-SA 3.0

Spheroidal mode $_0S_6^3$



https://saviot.cnrs.fr/index.en.html

Different earth models

- 1 D
  - PREM [Dziewonski & Andersion, 1981.](), PEPI, 25, 297
  - iasp91 Kennett & Engdal (1991), GJI, 105, 429
  - ak135 Kennett B.L.N., et al. (1995), GJI, 122, 108

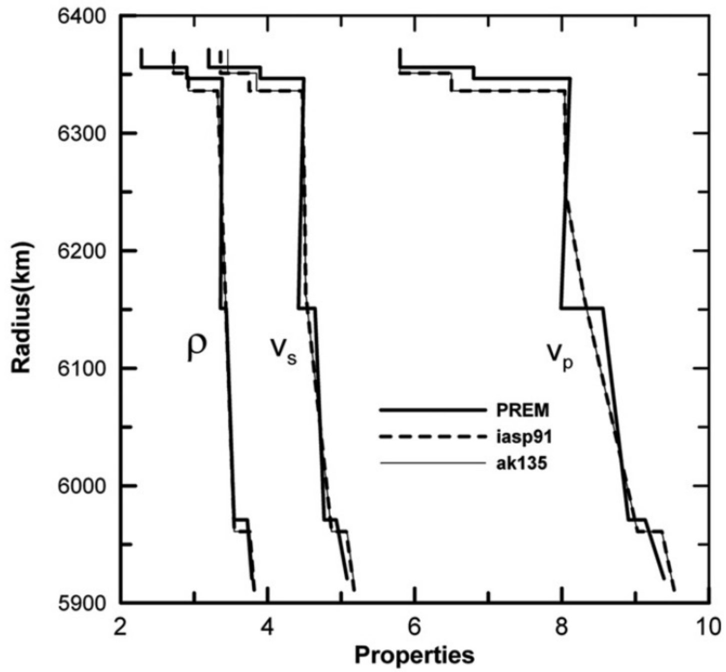


Fig. 1. Comparison of the density and velocity within 400 km depth among PREM, iasp91, and ak135. ρ—density; V_P—P-wave velocity and V_S–S-wave velocity.

Wang et al. (2012), Comput. Geosci, 49, 190

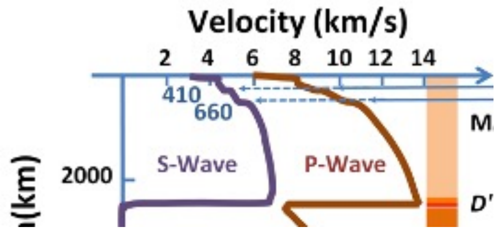
Resumé 3

- Cauchy momentum equation of the earth describes loading
  - No inertia forces
  - Gravity potentials due to surface mass and mass reconfiguration are considered
- Load love numbers depend on elasticity and density structure of the solid earth, so, on the earth model
- Structure of the earth model has to be determined from inversion of seismic waves and free oscillations

$$\nabla \cdot \boldsymbol{t} - \rho_0 \nabla \phi_1 + \nabla \cdot$$

$$\nabla^2 \phi_1 + 4\pi G \nabla \cdot (\rho_0$$

$$\boldsymbol{t} = \lambda \nabla \cdot \boldsymbol{u} + \mu (\nabla \boldsymbol{u}$$



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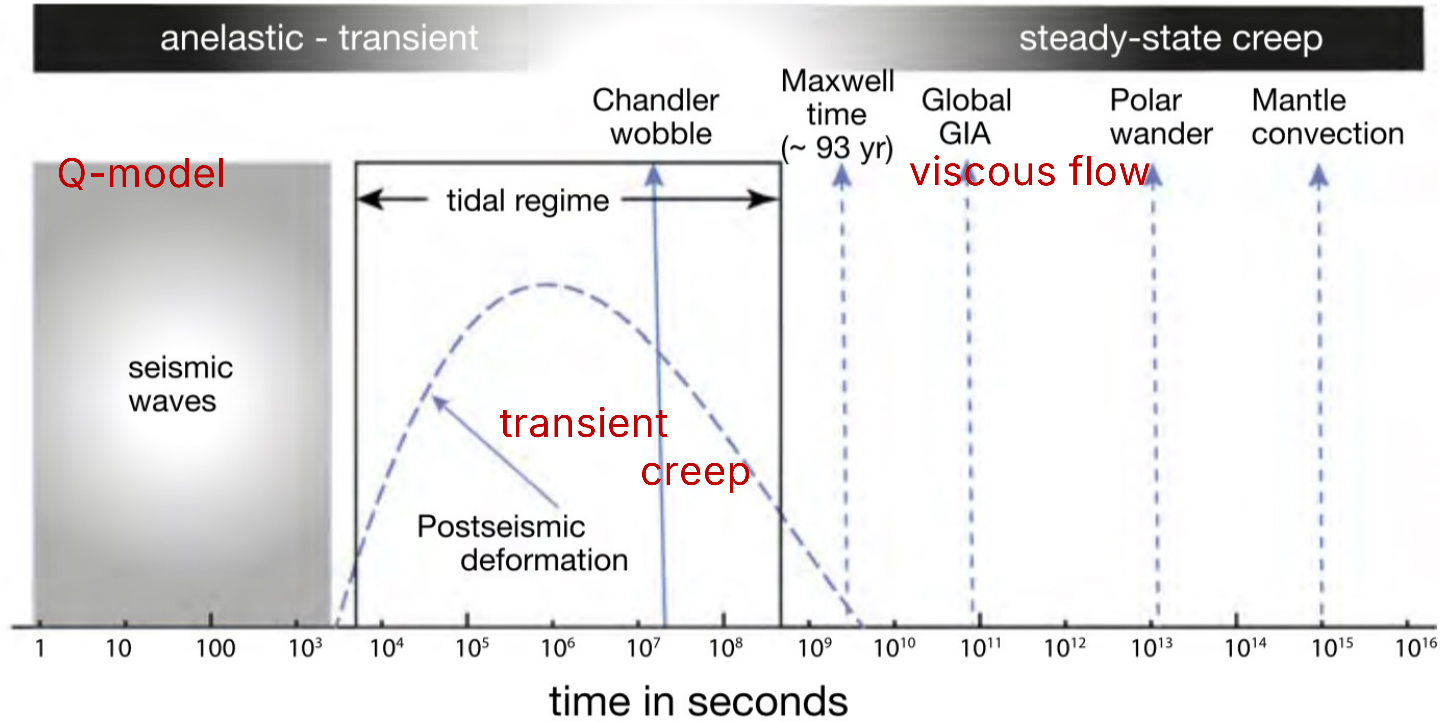
- Attenuation as a bridge between elastic and viscous behaviour



Ivins et al. 2020. Rep. Prog. Phys., 10, 106801

Time scale / spatial scale of forcing

- Instantaneous -> elastic response
- periodic -> elastic to anelastic response
- Secular -> anelastic to viscoelastic response

Maxwell rheology (viscoelasticity)

- Linear transition from elastic to viscous material behavior

$$\dot{\boldsymbol{\tau}} + \frac{\mu_0}{\eta}\boldsymbol{\tau} = 2\,\mu_0\,\dot{\boldsymbol{\epsilon}}^d$$

with Maxwell time $t_m = \frac{\eta}{\mu_0}$

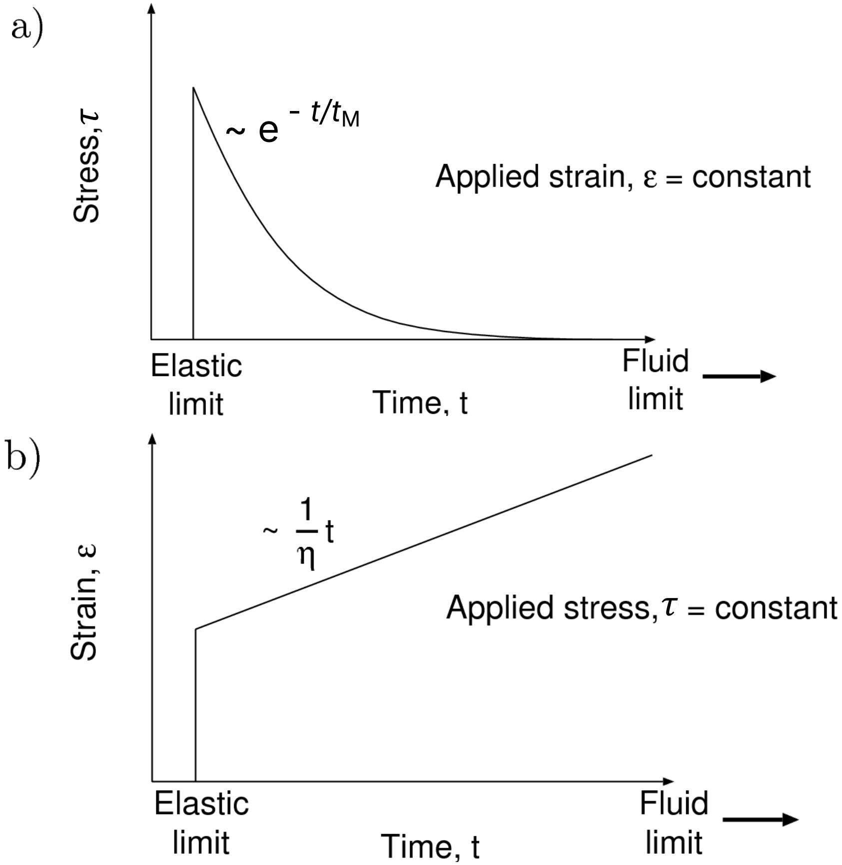
– the deviatoric stress $\boldsymbol{\tau}$

– the deviatoric strain $\boldsymbol{\epsilon}^d$.

Transformation into Laplace domain

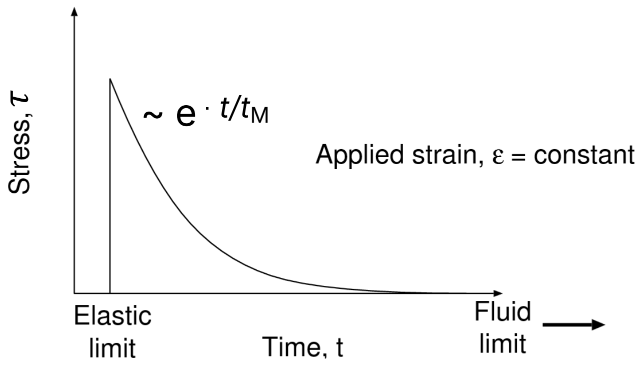
$$\boldsymbol{\tau}(s) = \mu(s)\,\boldsymbol{\epsilon}^d(s)$$

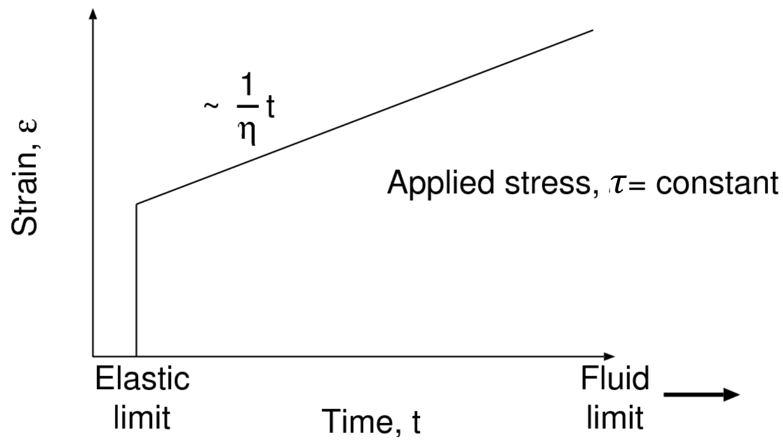
$$\mu(s) = \frac{2\,\mu_0\,s}{s + \mu_0/\eta}$$



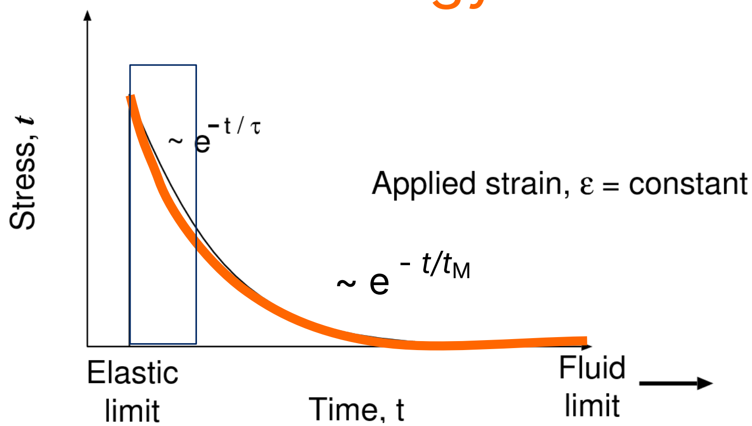
Steady state creep

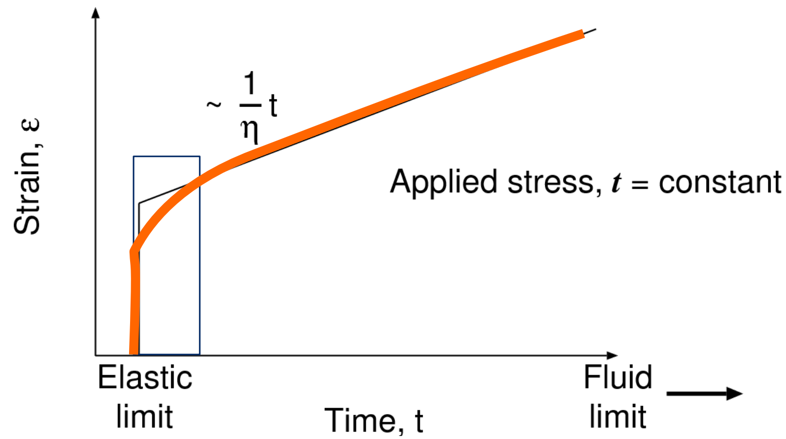
- Maxwell rheology (viscoelasticity)





- Transient rheology



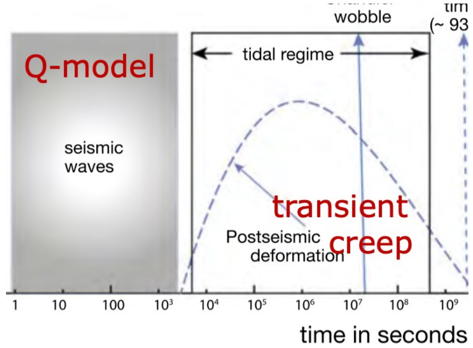


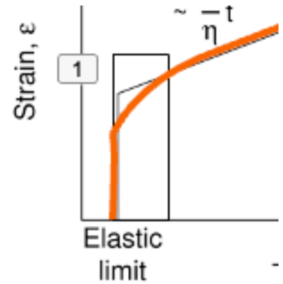
Mantle rheology

- lithosphere/asthenosphere at timescales from hours to decades
  - transient rheologies (e.g. Burger's)
  - Andrade model — Lau & Holtzman 2019. GRL, 46, 9544
  - extended Burger's rheology — Ivins et al. 2020. Rep. Prog. Phys., 10, 106801
- Can a single rheological model link these time scales?
  - Is this necessary, or can it be parametrised by a combination?
  - What is the role of power-law rheology here?

Resumé 4

- Anelasticity describes attenuation of seismic waves (1 to 10^3 s).
- Tidal frequencies are beyond this band (5×10^3 s to 5×10^6 s).
- Rheological model has to be extended.
- Anelasticity leads to reduction of shear modulus
- Post-seismic deformations
  - overlap with tidal frequencies,
  - but a transient rheology has to be considered for such a process.
- Goal is to find a rheology linking the seismic band and steady state creep and loading can help here.





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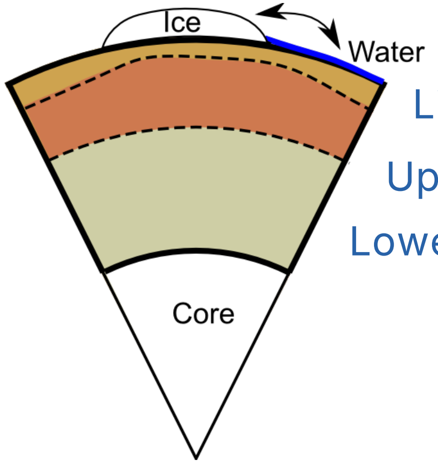
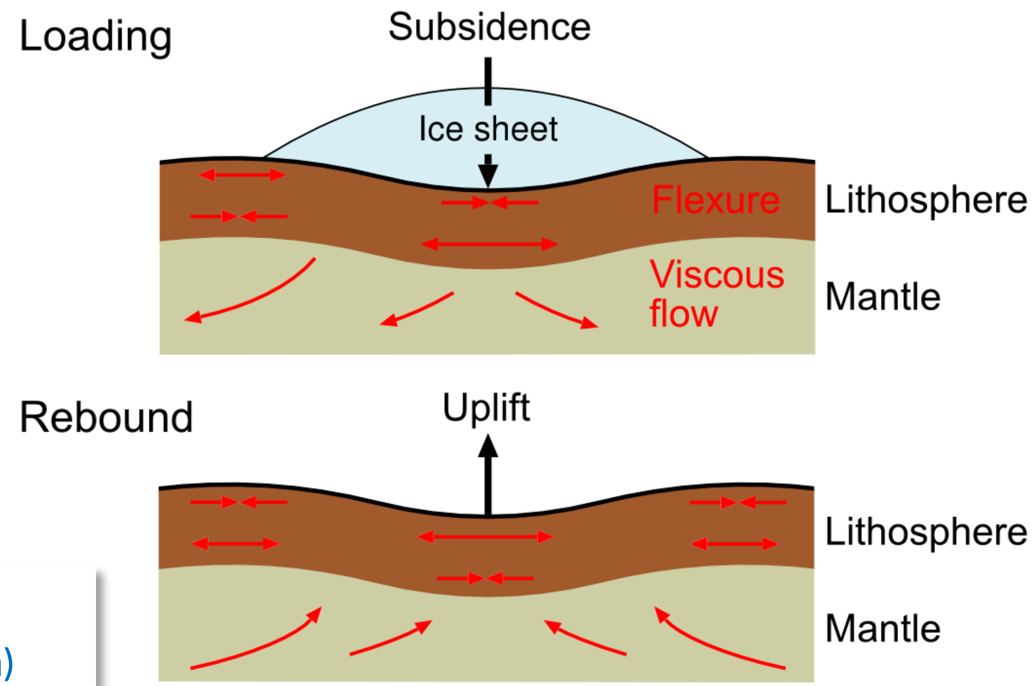




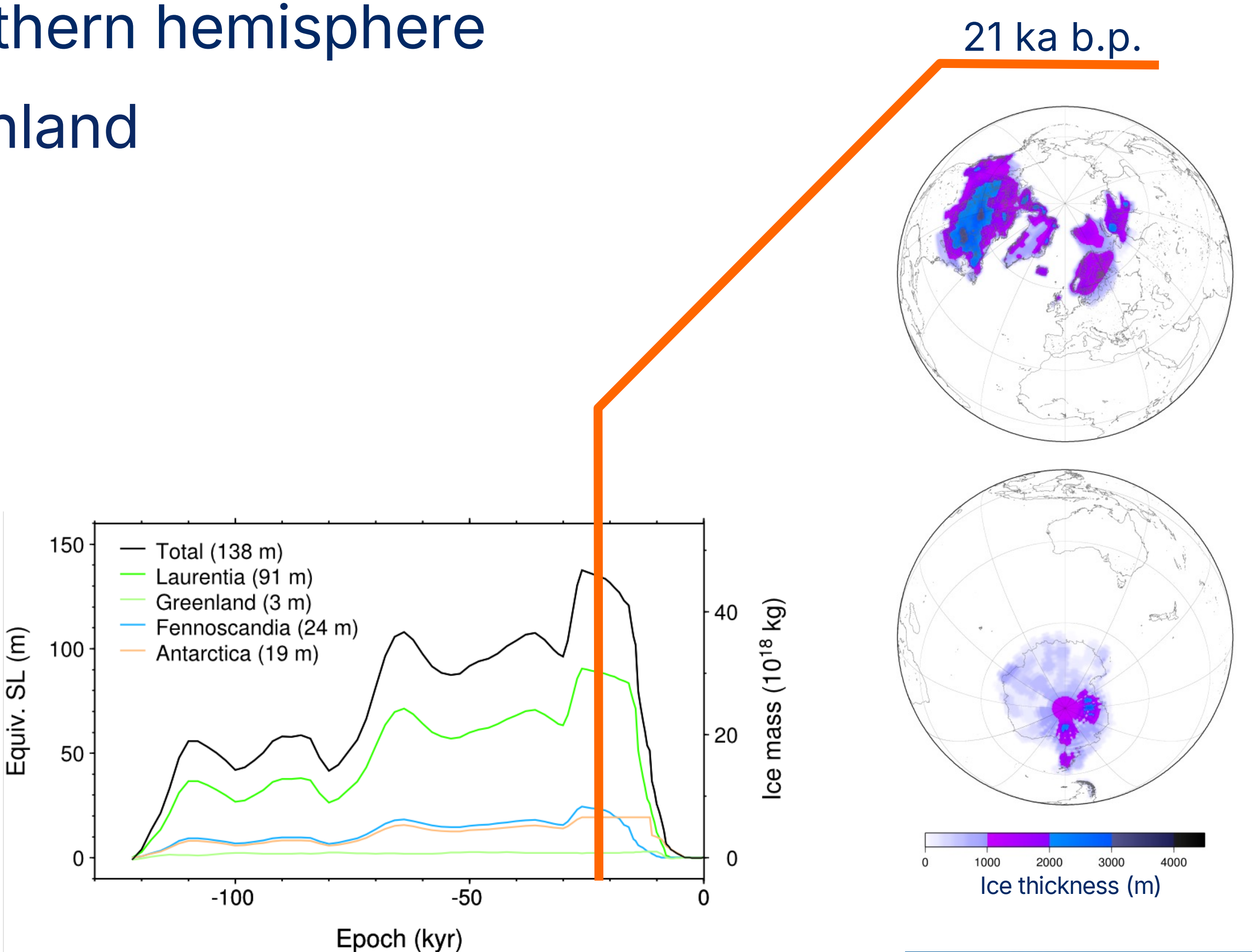
- load – (flexure + buoyancy+drag) = 0
- Continuum mechanics
  - elasticity
  - viscosity
  - gravity

Dimensions

- Extension (1000 km)
- Thickness (1000 m)
- Duration (10000 yr)

Last glacial cycle

- Glaciated regions on northern hemisphere
  - North America, Greenland
  - Fennoscandia
- On southern hemisphere
  - Antarctica

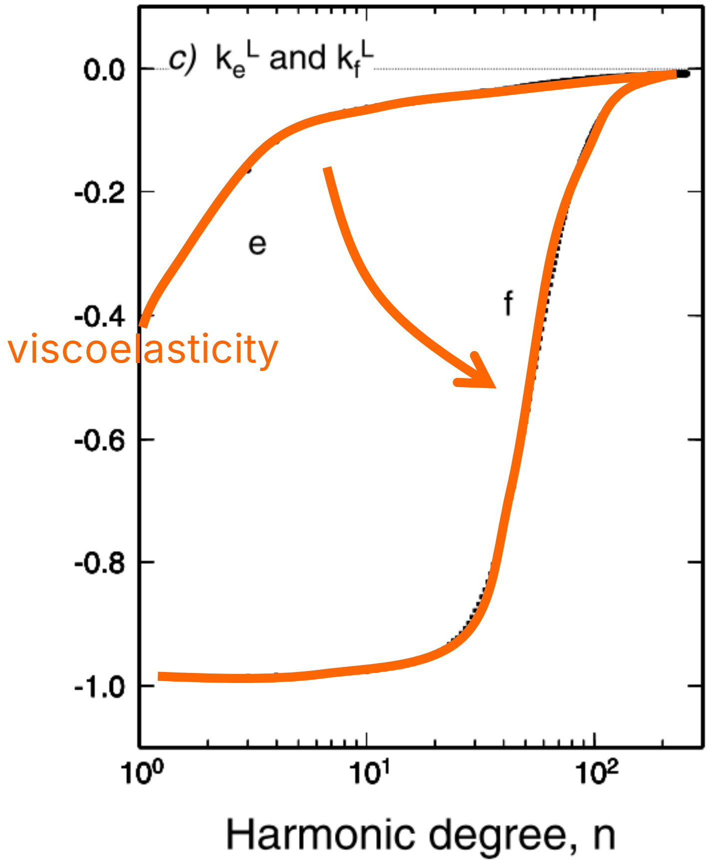


courtesy J.M. Hagedoorn

Time dependency of LLNs

$1 + k_l$

- Usual LLNs, k_e , of an elastic earth
- LLNs of flexing elastic lithosphere above a fluid mantle, k_f
  - n < 10 : isostatic equilibrium
  - 10 < n < 100 : flexure of an elastic plate
  - 200 < n : deformation of elastic body
- Viscoelasticity describes transition
  - governed by material flow in the earth mantle
  - solve the same equations but now for a viscoelastic continuum



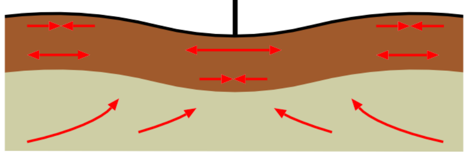
Spada et al. 2011. GJI, 185, 106

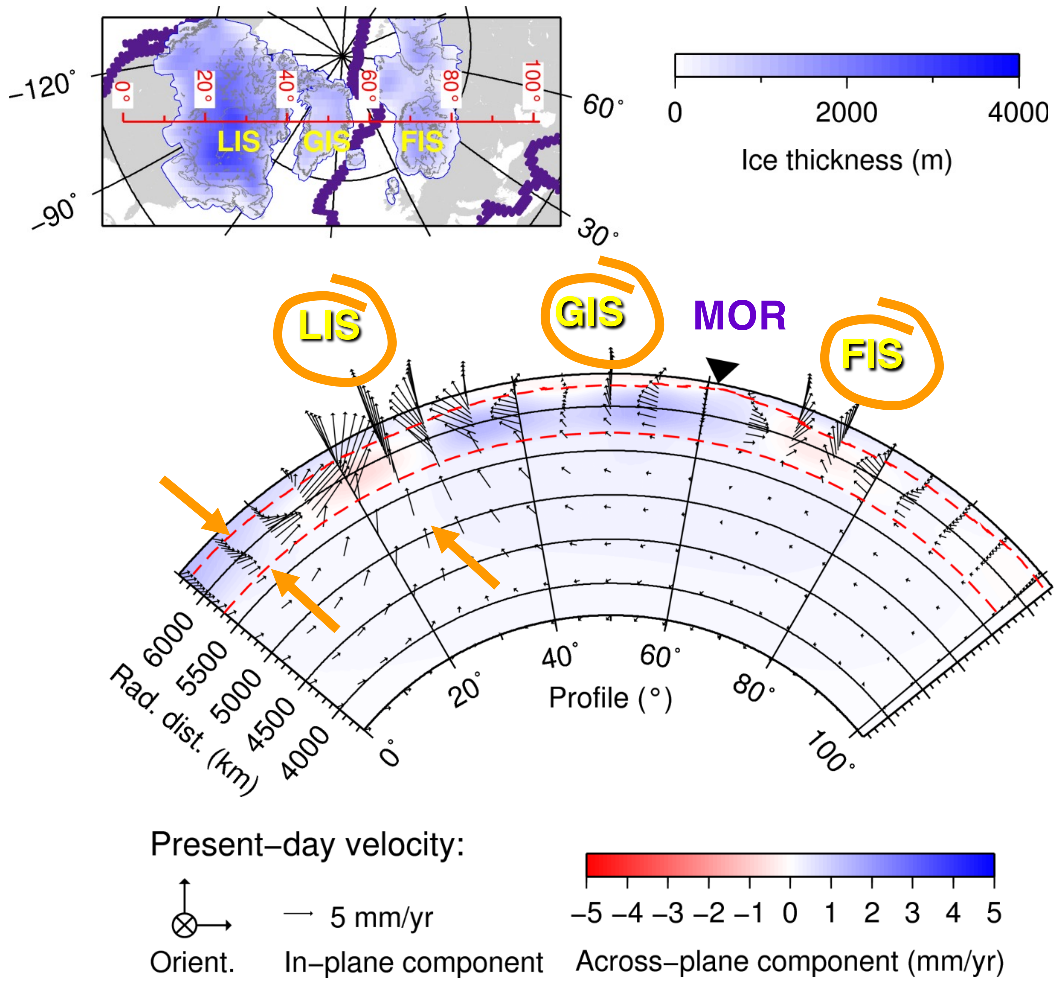
Cross section

Mid Ocean Ridge (**MOR**)

Ice sheets:

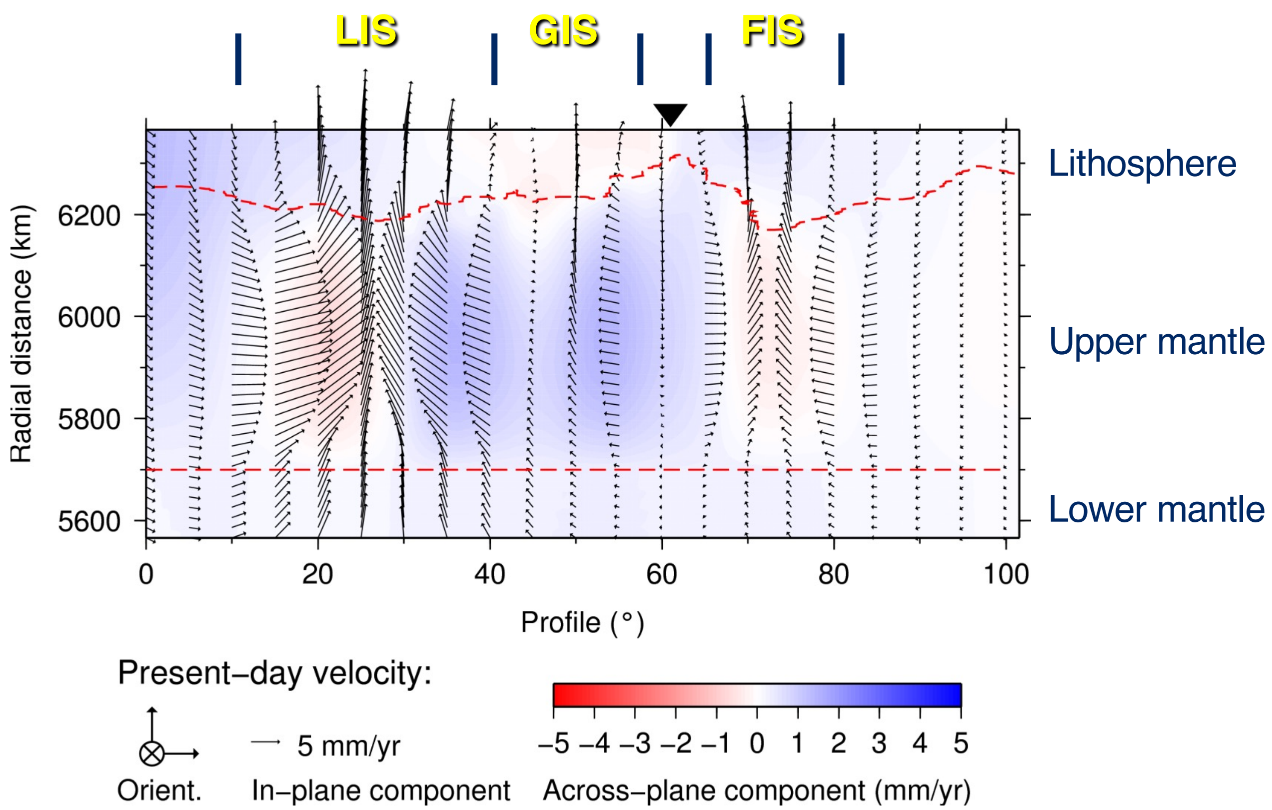
Laurentide (LIS) Greenland (GIS) Fennoscandia (FIS)





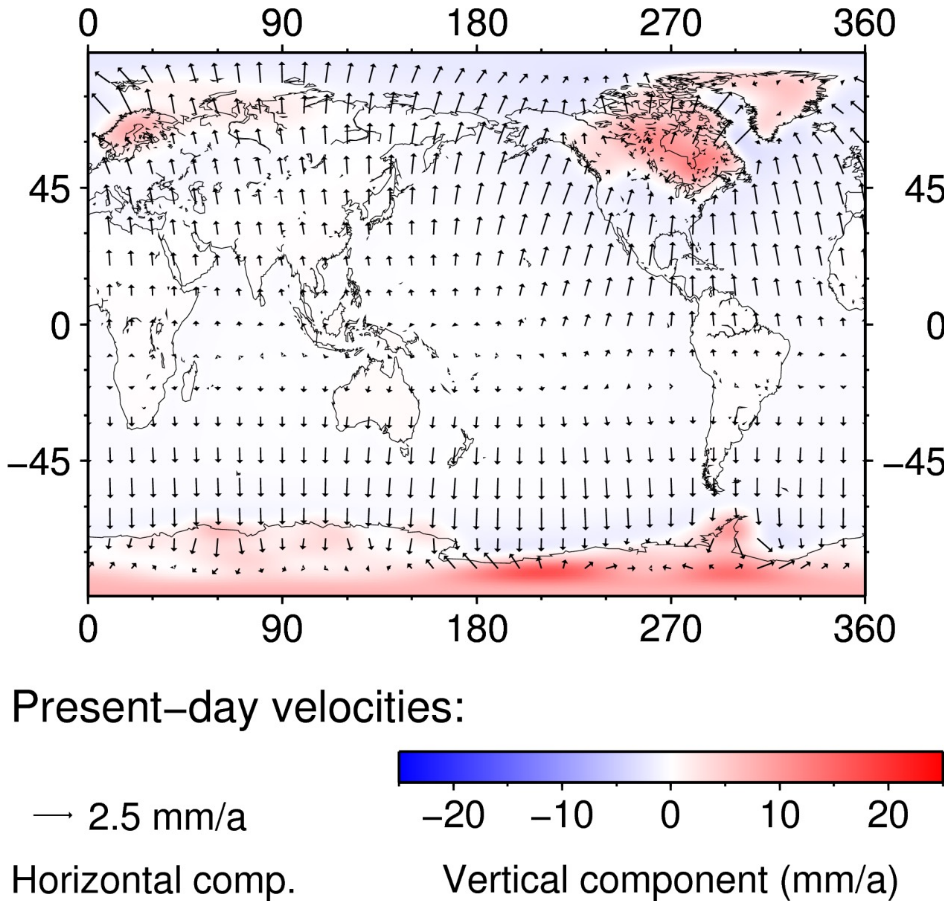
Klemann et al. 2008. J. Geodyn., 46, 159.

Cross section I (zoom)

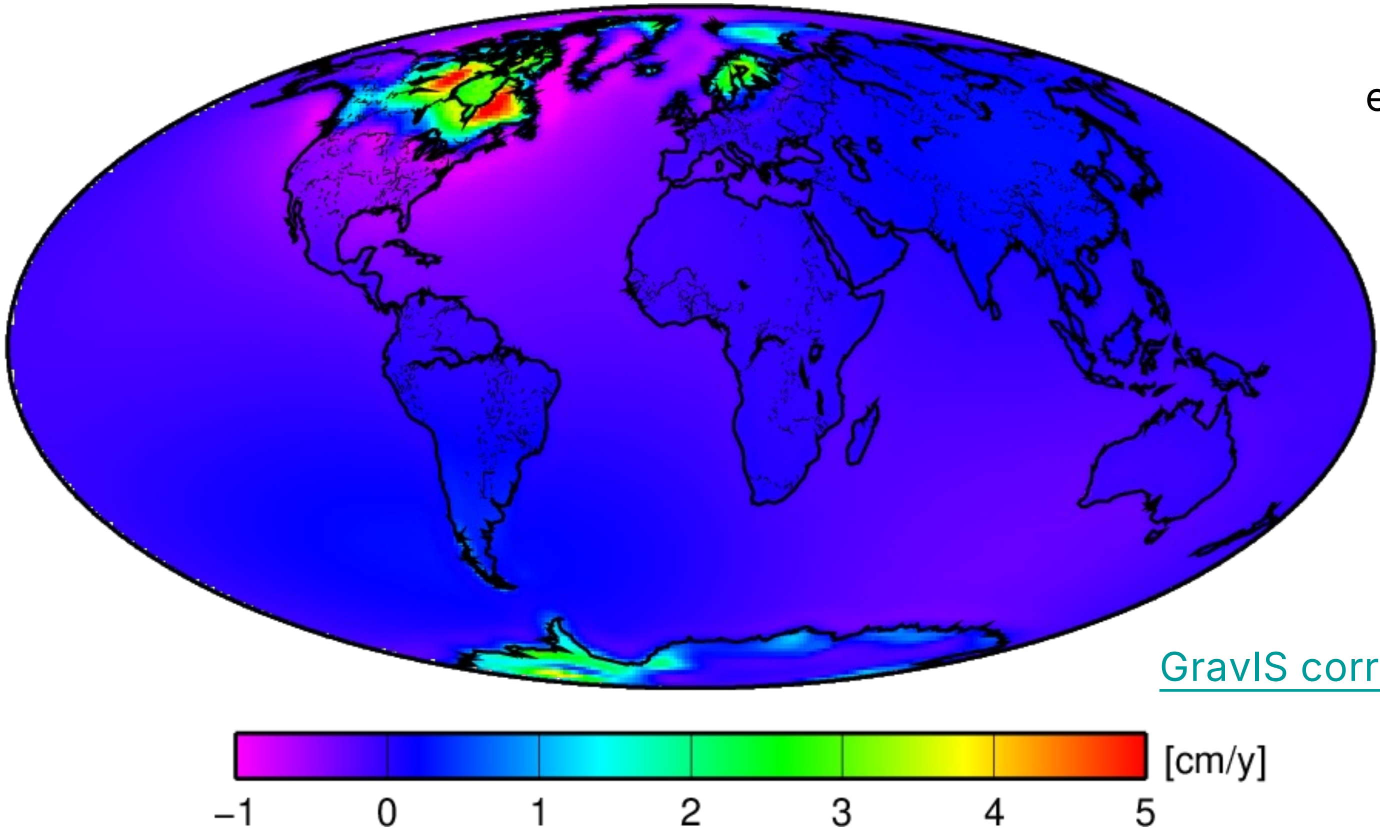


Displacement pattern

- Vertical displacement rate
  - order of 10 mm/a
  - confined to formerly glaciated regions
- Horizontal displacement rate
  - order of 1 mm/a
  - directed towards formerly glaciated regions



Effect on gravity field



GIA correction due to ICE6G expressed as equivalent water height

ICE6G/VM5 is a glaciation history inferred from glaciological, geological and geodetic data

GravIS correction

Rem.: Viscosity structure VM5 is inferred in combination with ICE6G

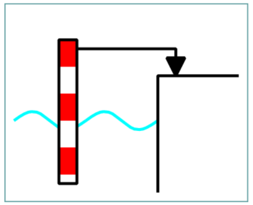
Sea level

The geoid is defined as

$$n(\Omega, t) := e(\Omega, t) + h_{\mathrm{wl}}(t)$$

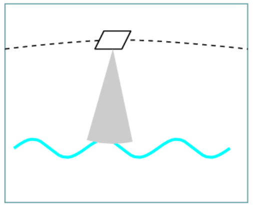
with h_{wl} the distance between the reference-potential height and the potential height which the current sea level is following.

Then, relative sea level:



$$h_{\mathrm{RSL}}(\Omega, t) := [n-u](\Omega, t) - [n-u](\Omega, t_0) ,$$

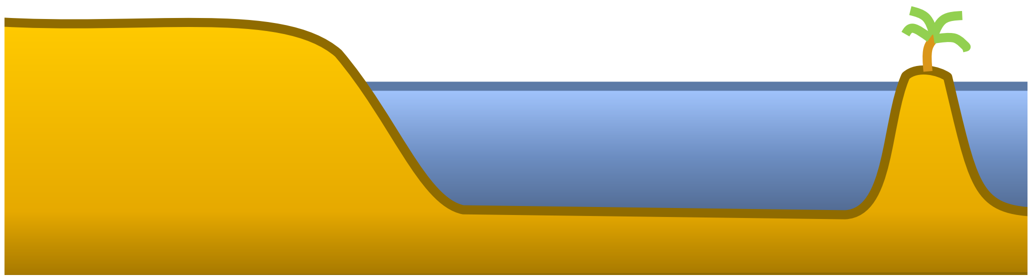
altimetric sea level:



$$h_{\mathrm{alt}}(\Omega, t) := n(\Omega, t) - n(\Omega, t_0)$$

Sea-level equation

1. **Unperturbed state**
2. Ice load
3. Conservation of mass = water + ice
4. Change of geoid due to ice mass
5. Solid-earth deformation
6. Change of geoid due to solid earth (glacial maximum)
7. Glacial maximum with and without sea level equation (pt. 6 – pt. 3)
8. Melting of ice: conservation of mass + change of geoid
9. Solid-earth deformation (present day)
10. Difference to unperturbed state (pt. 9 – pt. 1)



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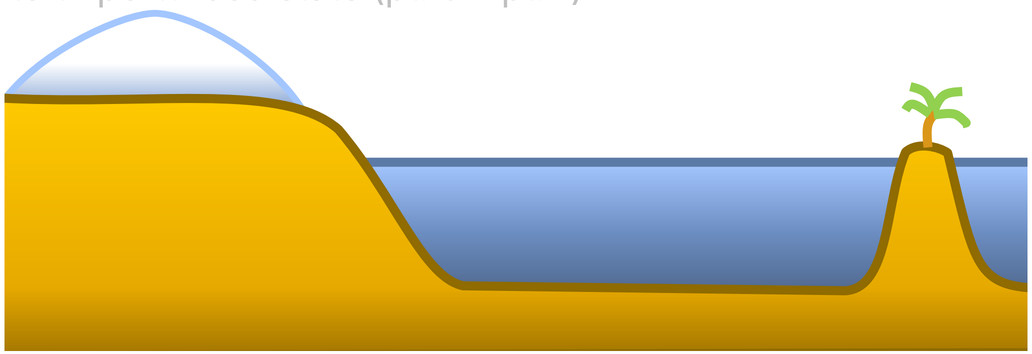

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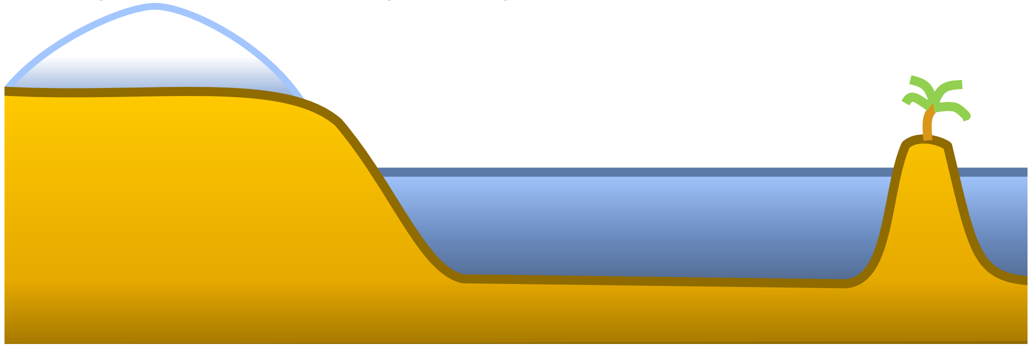

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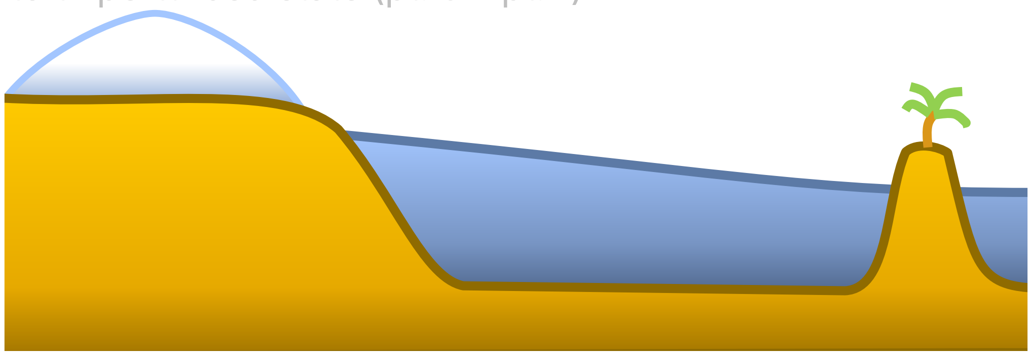

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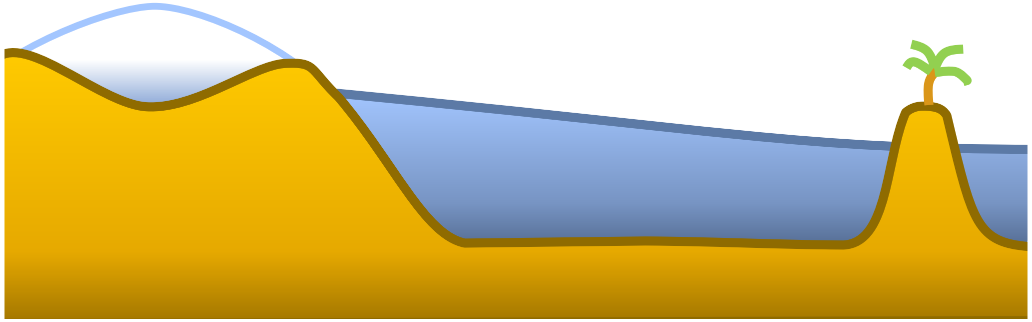

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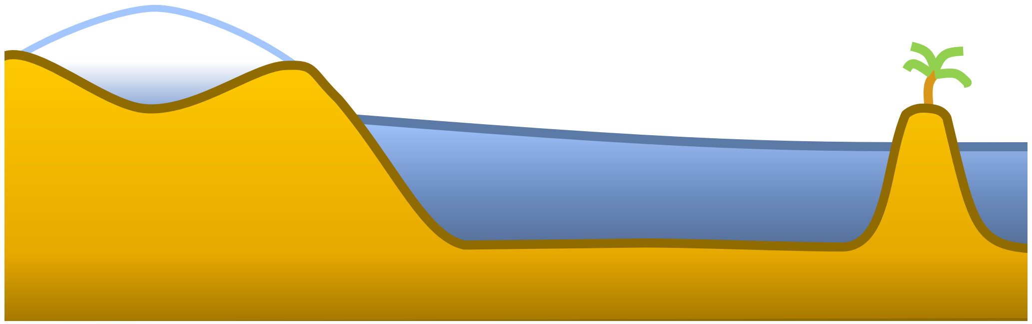

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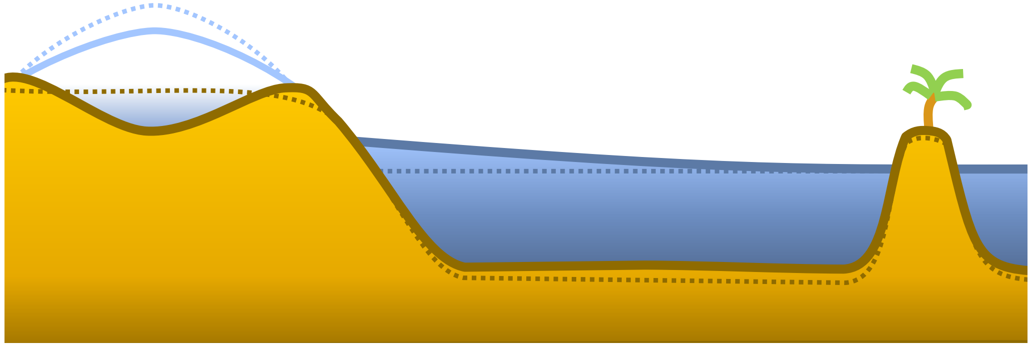

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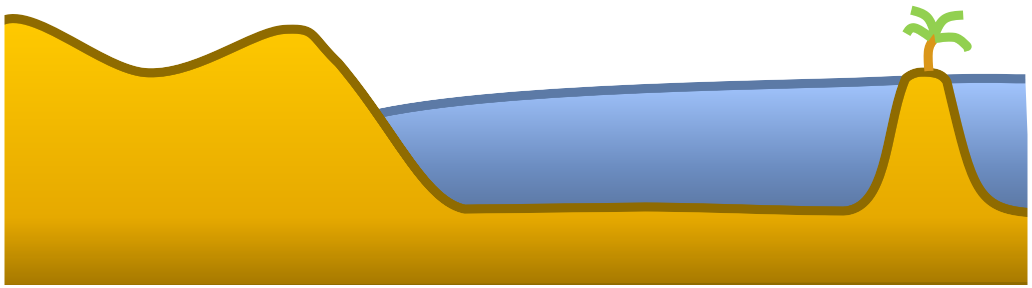

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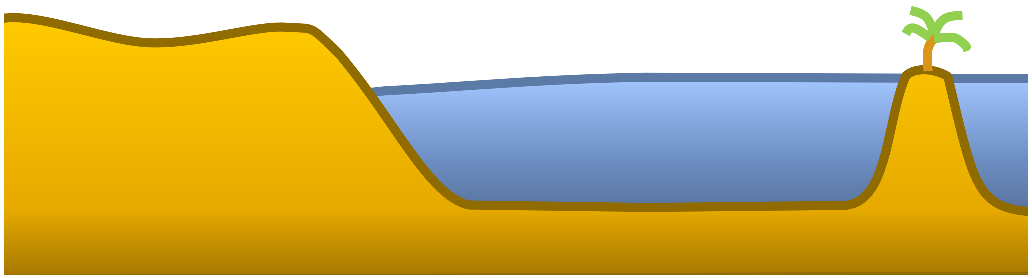

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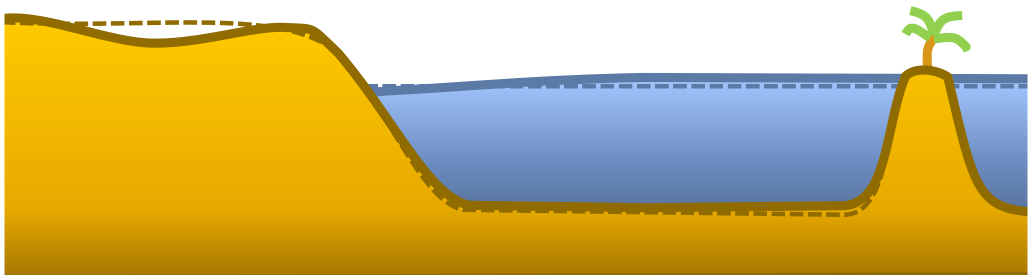
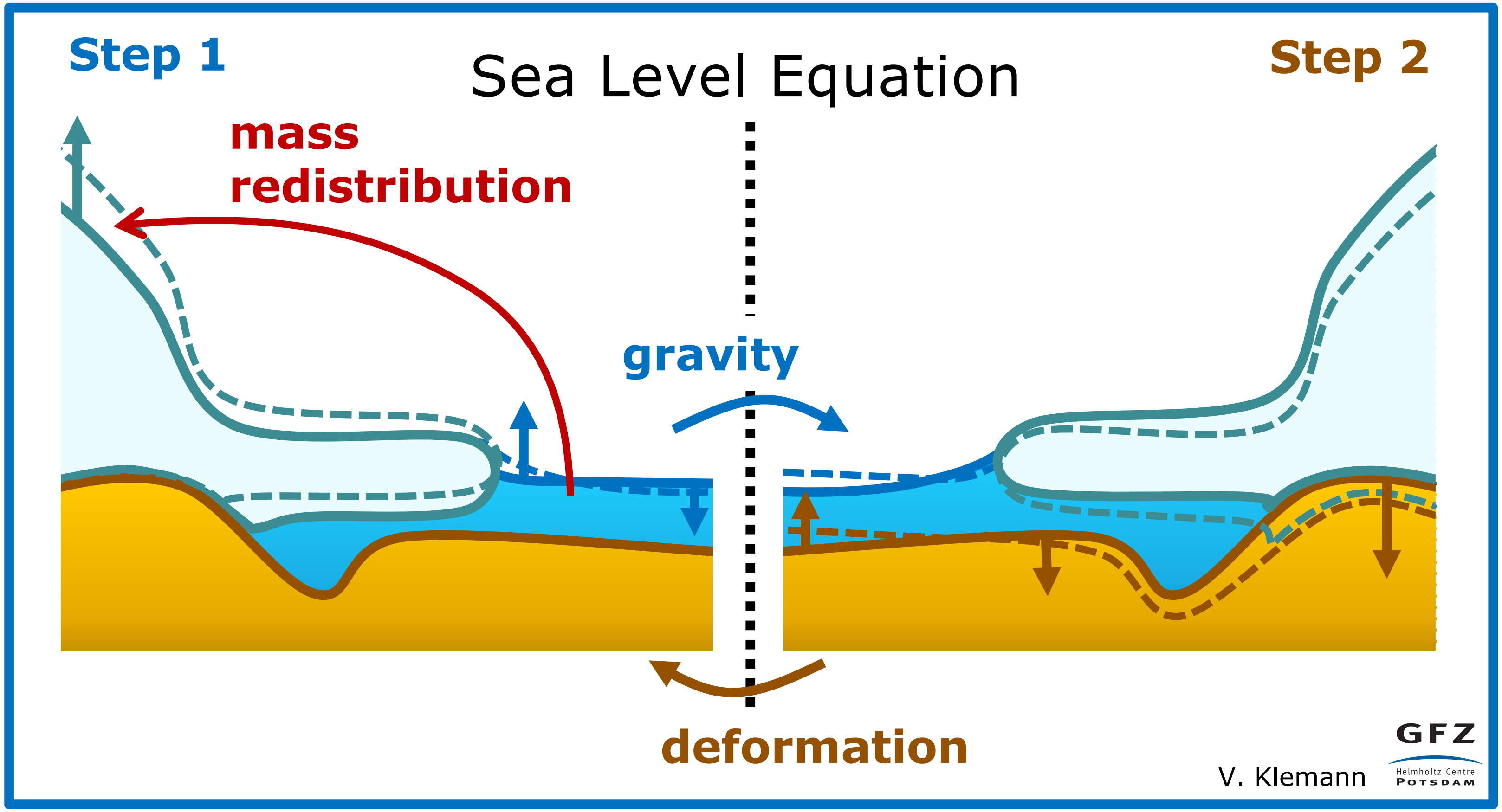
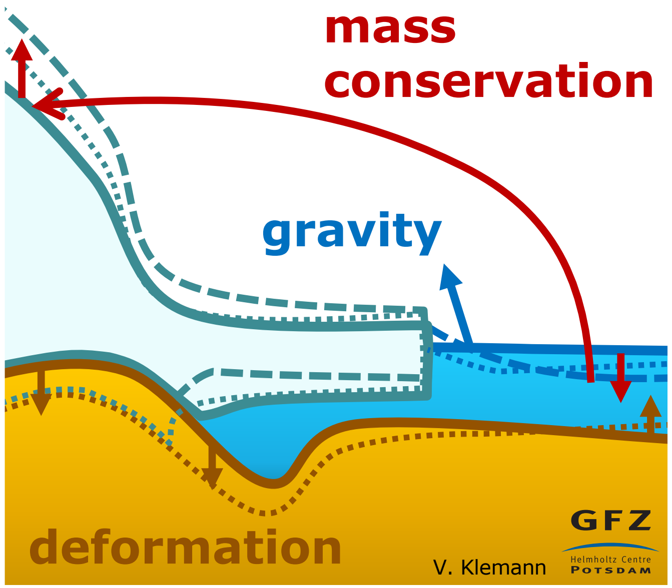


Effect of gravity and deformation on sea-level change



Effect of gravity and deformation on sea-level change

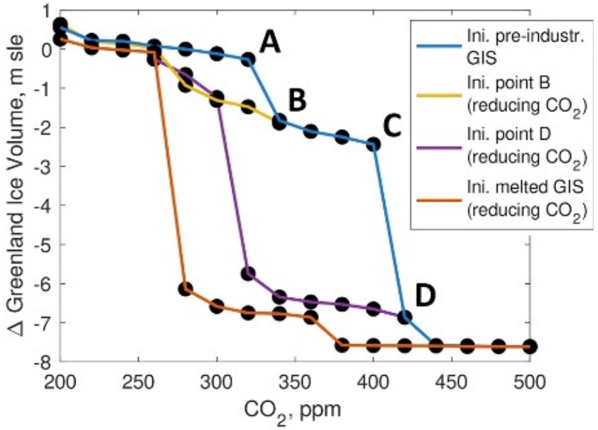
Sea-Level Equation

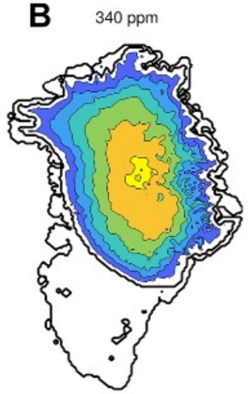


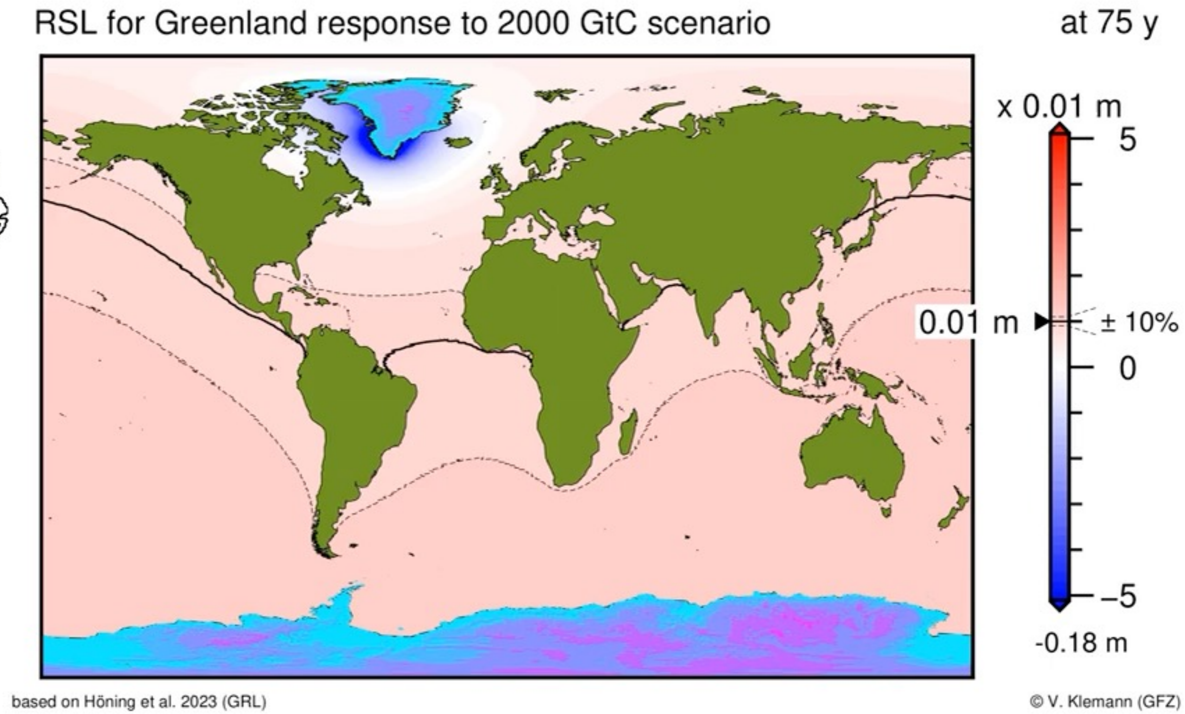
1. Mass transport between ice and ocean is conservative
2. Gravity potential defines ocean surface
3. Solid earth deforms due to surface loading
4. Solution has to be determined iteratively

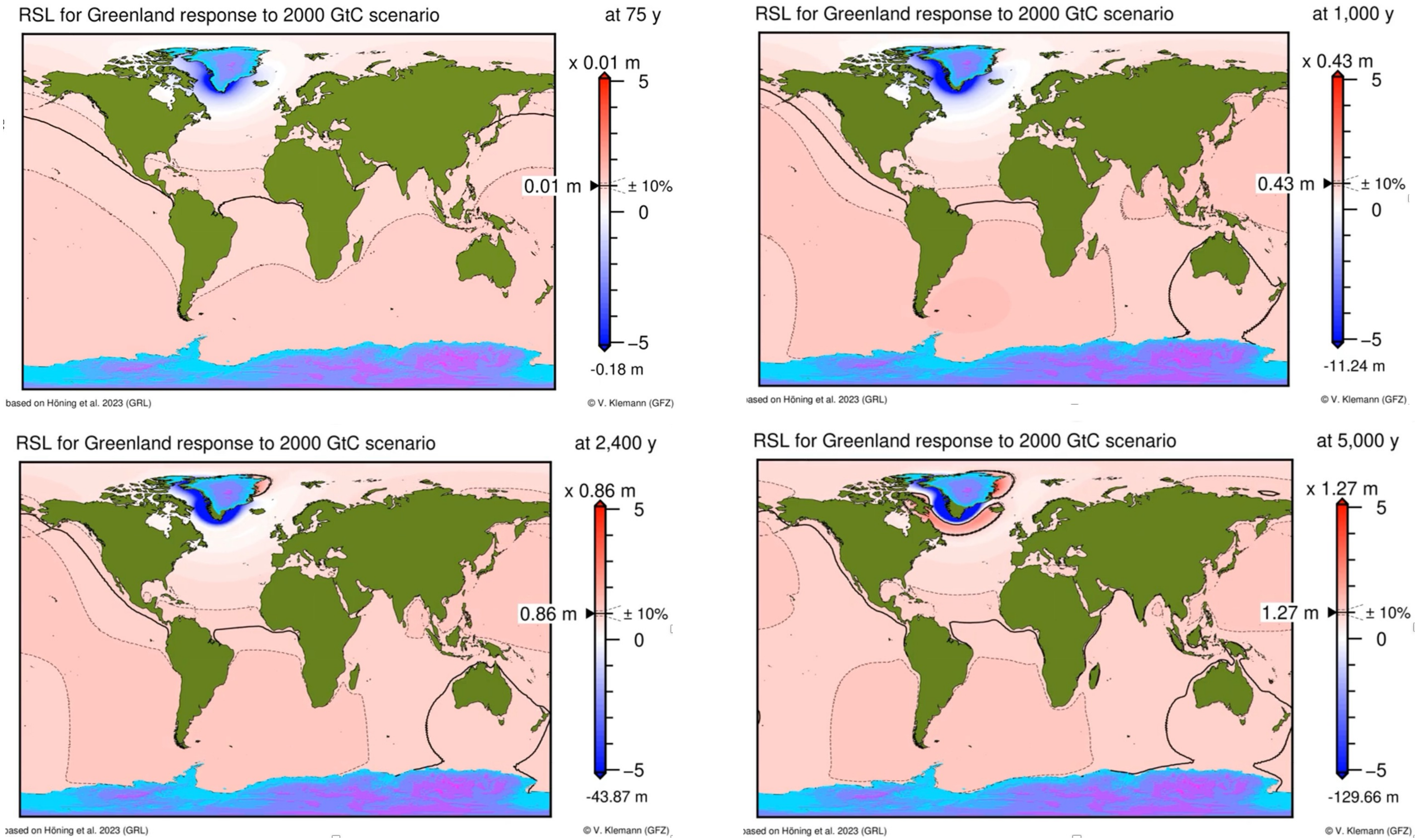
Fingerprint

- **Long-term fingerprints for Greenland melting event extracted from ([Höning et al., 2023, GRL](#))**



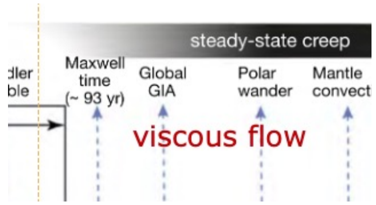




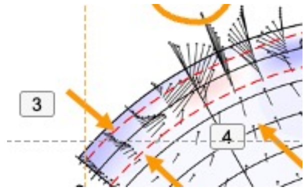

RSL for Greenland response to 2000 GtC scenario
at 75 y
x 0.01 m
5
0.01 m
± 10%
0
−5
-0.18 m
based on Höning et al. 2023 (GRL)
© V. Klemann (GFZ)
RSL for Greenland response to 2000 GtC scenario
at 1,000 y
x 0.43 m
5
0.43 m
± 10%
0
−5
-11.24 m
based on Höning et al. 2023 (GRL)
© V. Klemann (GFZ)
RSL for Greenland response to 2000 GtC scenario
at 2,400 y
x 0.86 m
5
0.86 m
± 10%
0
−5
-43.87 m
based on Höning et al. 2023 (GRL)
© V. Klemann (GFZ)
RSL for Greenland response to 2000 GtC scenario
at 5,000 y
x 1.27 m
5
1.27 m
± 10%
0
−5
-129.66 m
based on Höning et al. 2023 (GRL)
© V. Klemann (GFZ)

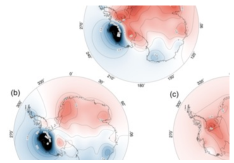
Resumé 5

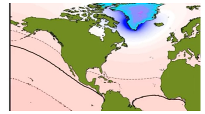
- GIA describes deformational response of the earth to glacial loads on time scales of yrs to thousands of yrs
- The Earth’s mantle behaviour changes on these time scales from that of an elastic body to that of a fluid
- Viscoelasticity describes the transition
- Time scale of the GIA process, the relaxation time, is governed by efficiency of material transport in the mantle
- GIA gravity change signal
  - mainly generated by ongoing uplift
  - pronounced signal in linear trends of GRACE(-FO)
- Big problem: how to separate glaciation history from viscosity structure
  - geodynamic constraints provide structures lateral heterogeneities
- GIA is generated by glacial cycles, due to an interplay between ice sheets, ocean and solid earth in a highly dynamic climate system.



$$\dot{\boldsymbol{\tau}} + \frac{\mu_0}{\eta}\boldsymbol{\tau} = 2\,\mu_0\,\dot{\boldsymbol{\epsilon}}^d$$







Further reading

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