NEROGRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

International Spring School
Neustadt an der Weinstraße, Germany, March 10-14, 2025

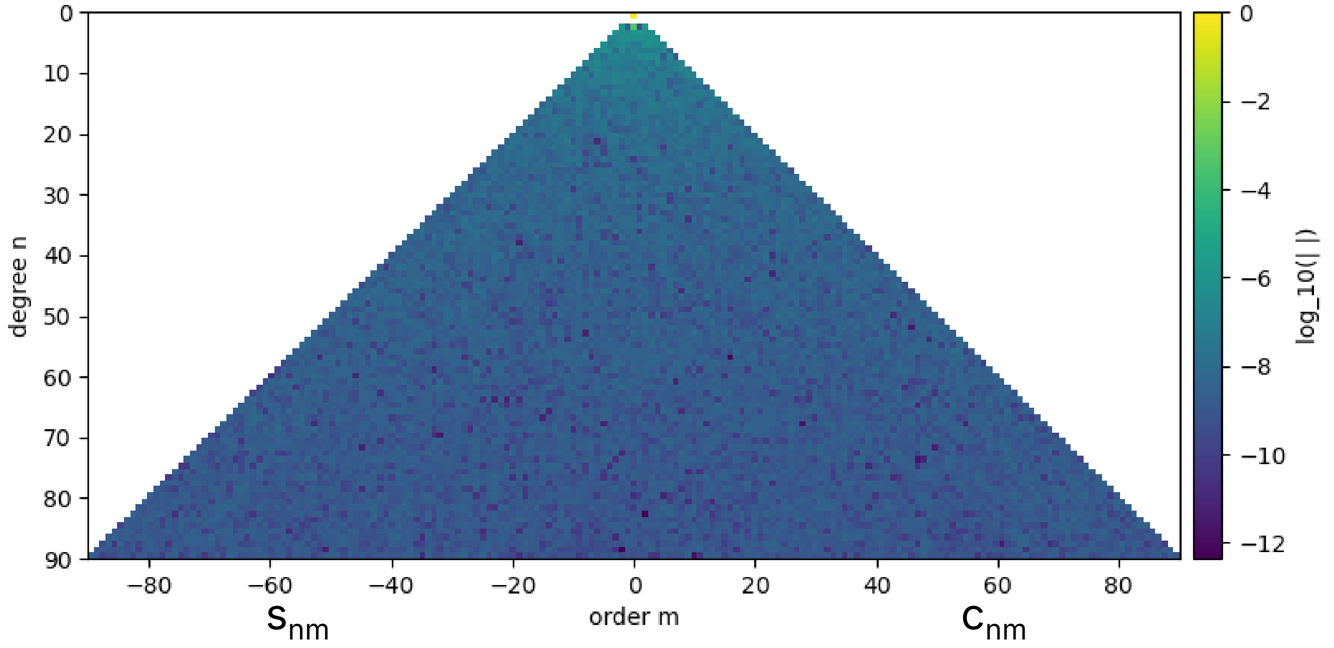
From Level-2 Spherical Harmonics to Level-3 Gridded Data

Eva Boergens
(GFZ Helmholtz Centre for Geosciences, boergens@gfz.de)

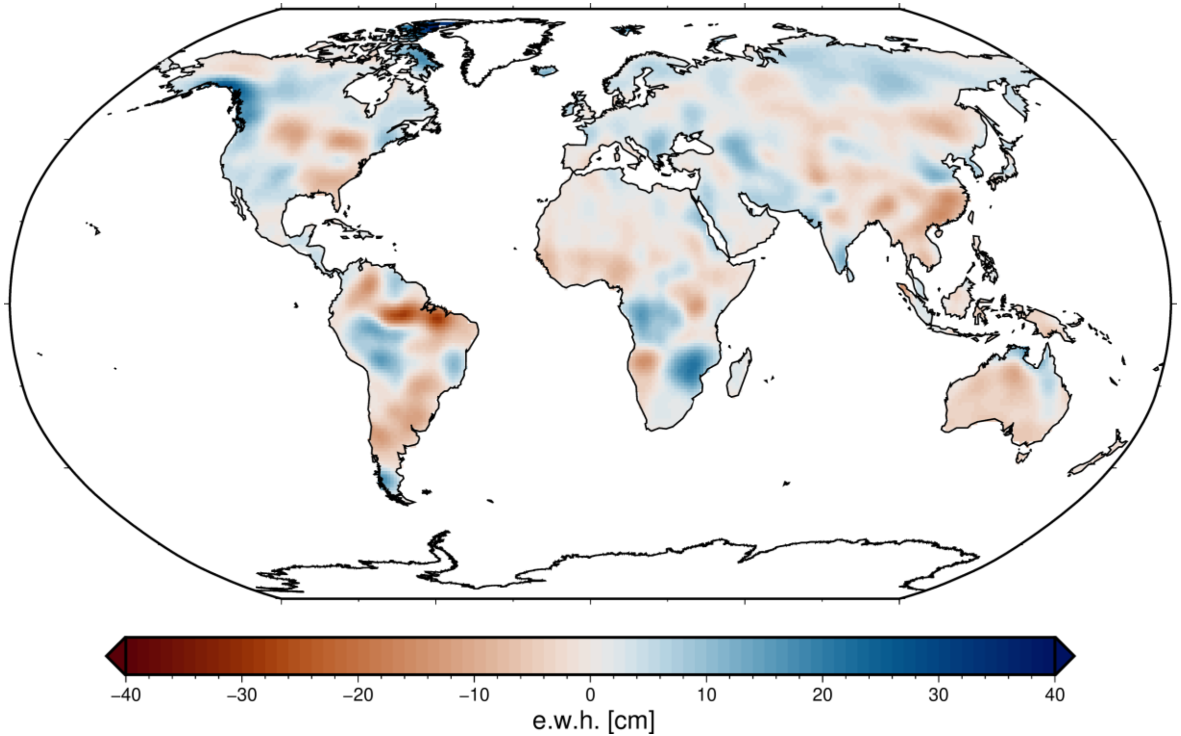
TU berlin · Freie Universität Berlin · Technische Universität München · GFZ Helmholtz Centre for Geosciences · Universität Bonn · hcu HafenCity Universität Hamburg

What do we have so far?

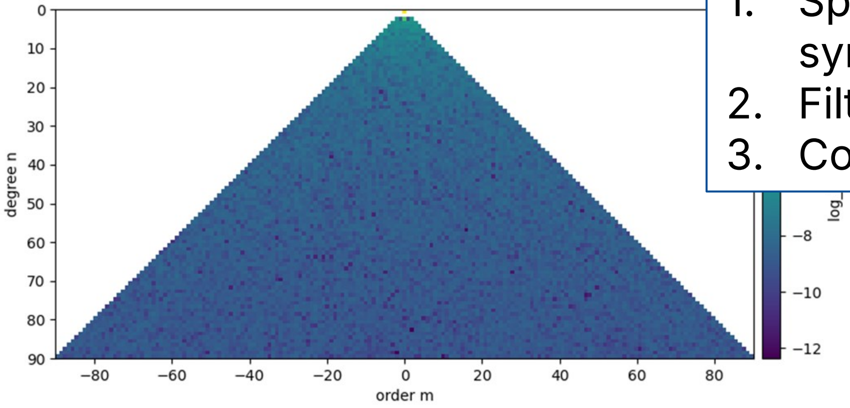
Level-2 data (lecture by T. Gruber on Monday)



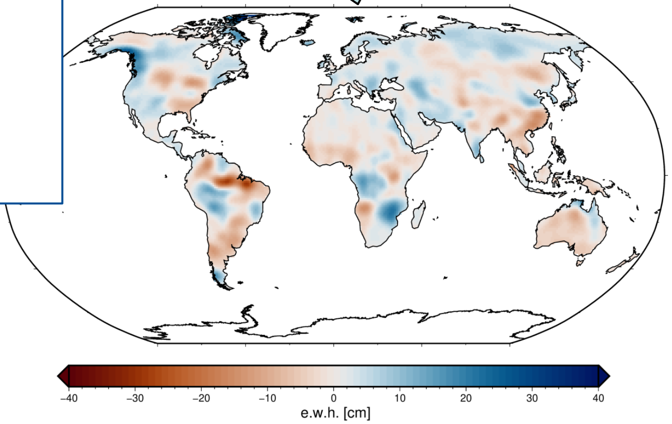
What do we want?



This lecture



1. Spherical harmonic synthesis
2. Filtering
3. Corrections



4. Leakage
5. Alternatives: Mascons

1. Spherical Harmonic Synthesis

Spherical Harmonic Synthesis

Equivalent water height

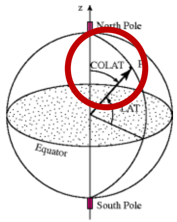
density water

Sum over all degrees and orders

Spherical harmonic (or Stokes) coefficients L2 data

$$ewh(\lambda, \theta)[m] = \frac{1}{\rho_W} \frac{M}{2\pi R^2} \sum_{n=0}^{N} \frac{2n+1}{1+k'_n} \sum_{m=0}^{n} (c_{nm} P_{nm}(\cos\theta) \cos m\lambda + s_{nm} P_{nm}(\cos\theta) \sin m\lambda)$$

longitude, co-latitude



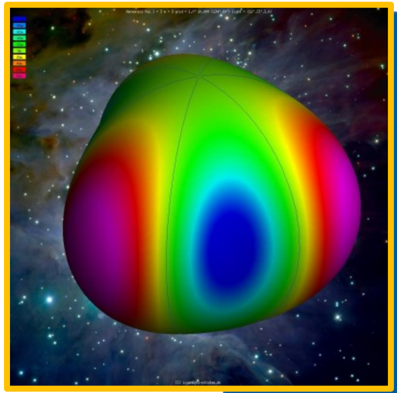
M: mass of Earth
R: Earth‘s radius

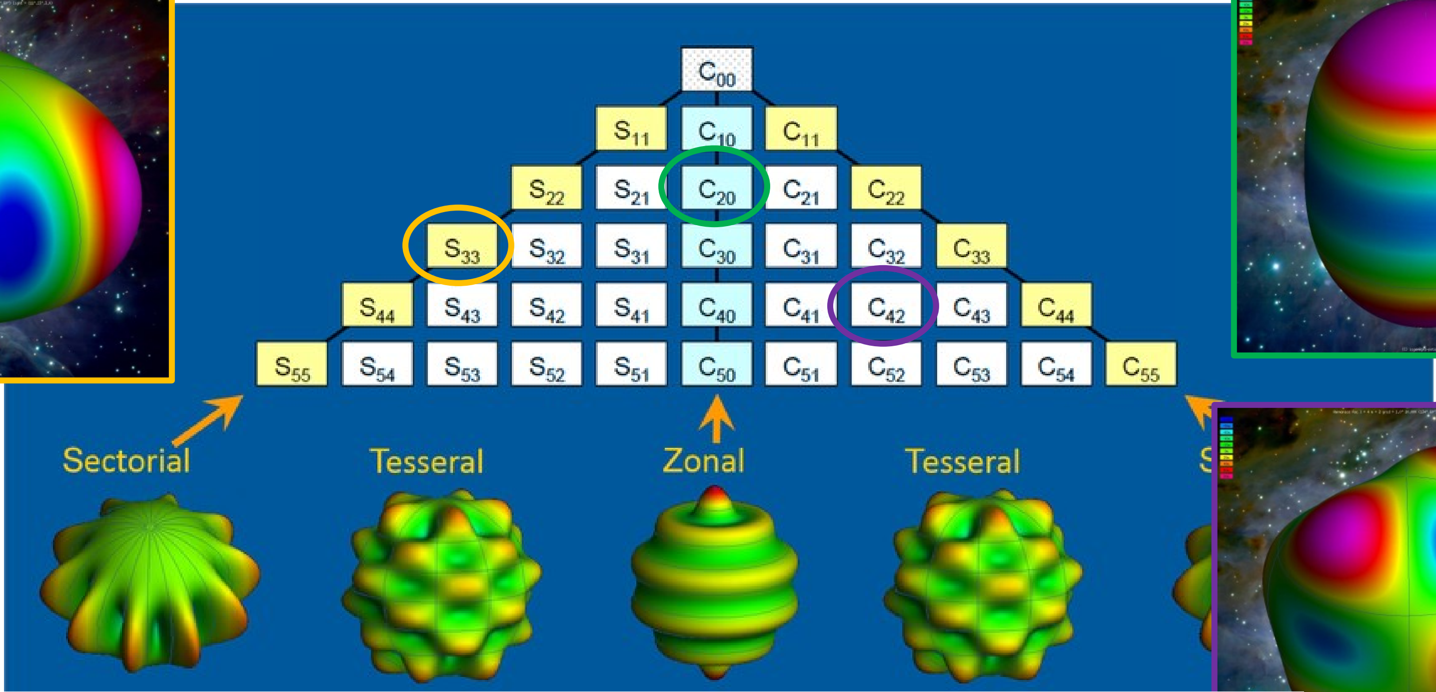
Load-Love-Number
Describing elastic deformation of the Earth

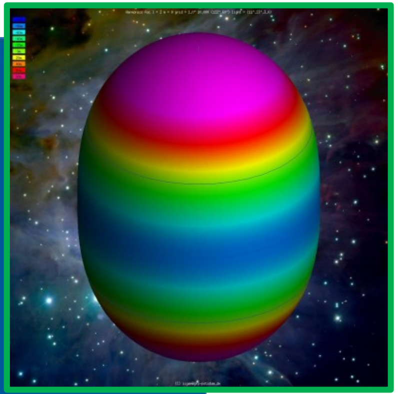
Basis functions
P_{nm}: fully normalised Legendre functions

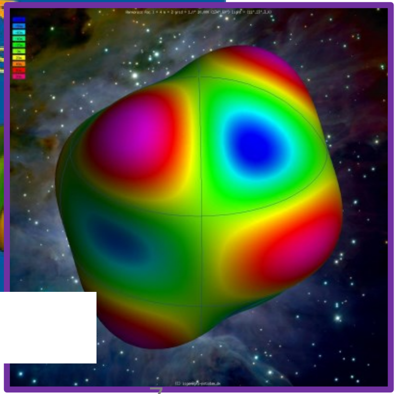
$$C_{nm} = P_{nm}(\cos\theta)\cos m\lambda$$
$$S_{nm} = P_{nm}(\cos\theta)\sin m\lambda$$

Recap: Spherical Harmonic Basis Functions





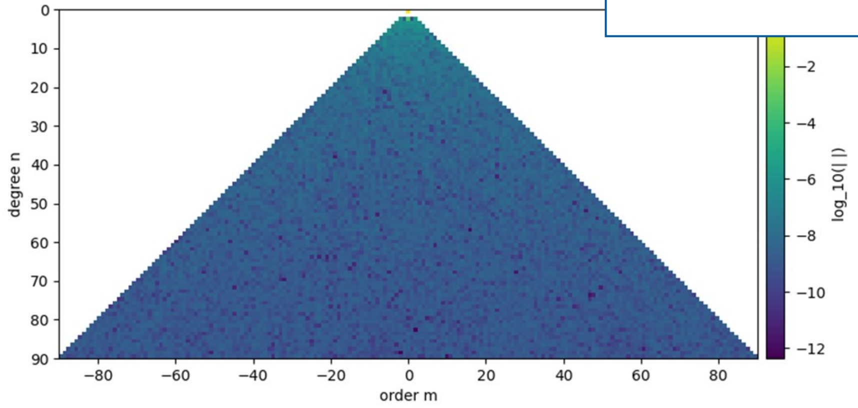


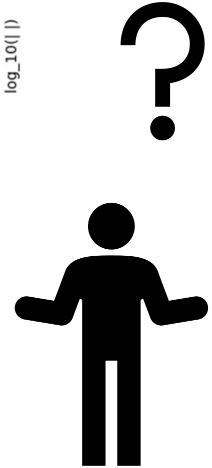


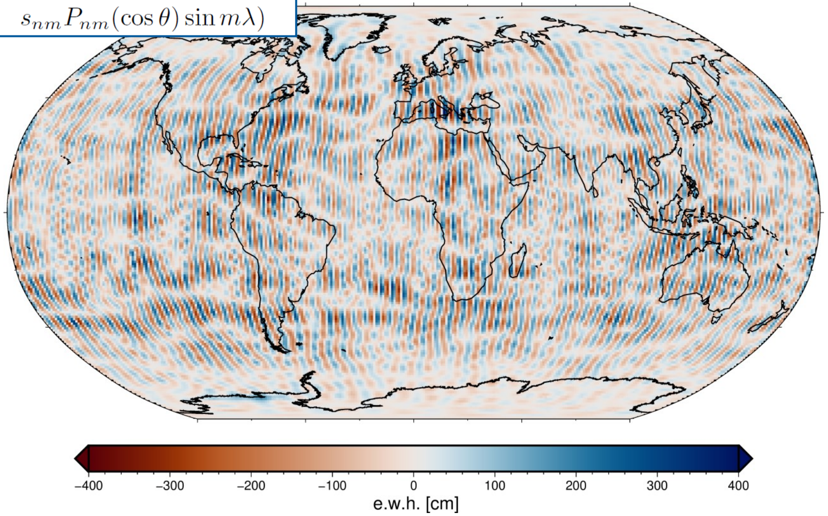
Visualisation: https://icgem.gfz-potsdam.de/vis3d/tutorial

Spherical Harmonic Synthesis

$$ewh(\lambda,\theta)[m] = \frac{1}{\rho_W}\frac{M}{2\pi R^2}\sum_{n=0}^{N}\frac{2n+1}{1+k'_n}\sum_{m=0}^{n}(c_{nm}P_{nm}(\cos\theta)\cos m\lambda + s_{nm}P_{nm}(\cos\theta)\sin m\lambda)$$

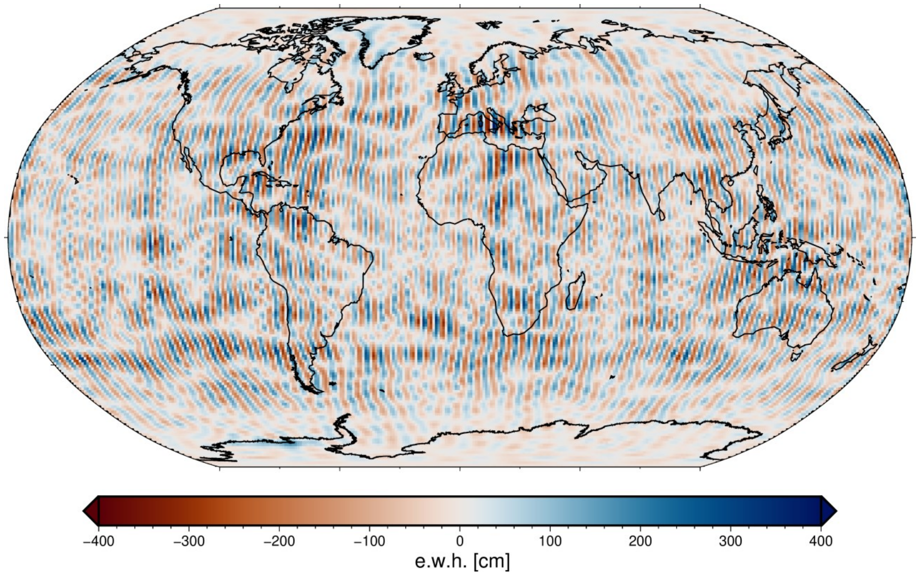






2. Filtering

Spatially Correlated Striping Errors



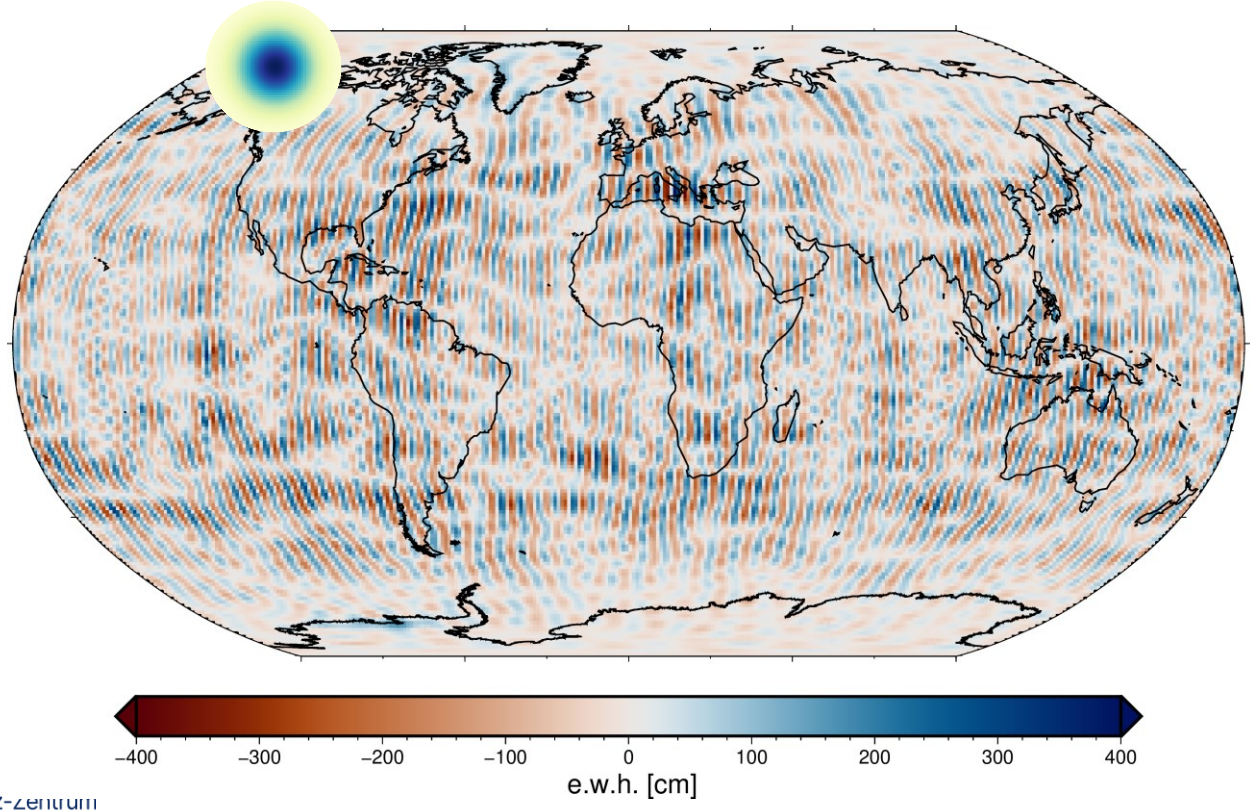
- Residuals of high frequency signals not covered by background models
- Near polar orbit of satellites leads to higher correlations along-track (North-South direction)

Solution: Filtering of gravity fields!

GFZ Helmholtz-Zentrum für Geoforschung

Filtering

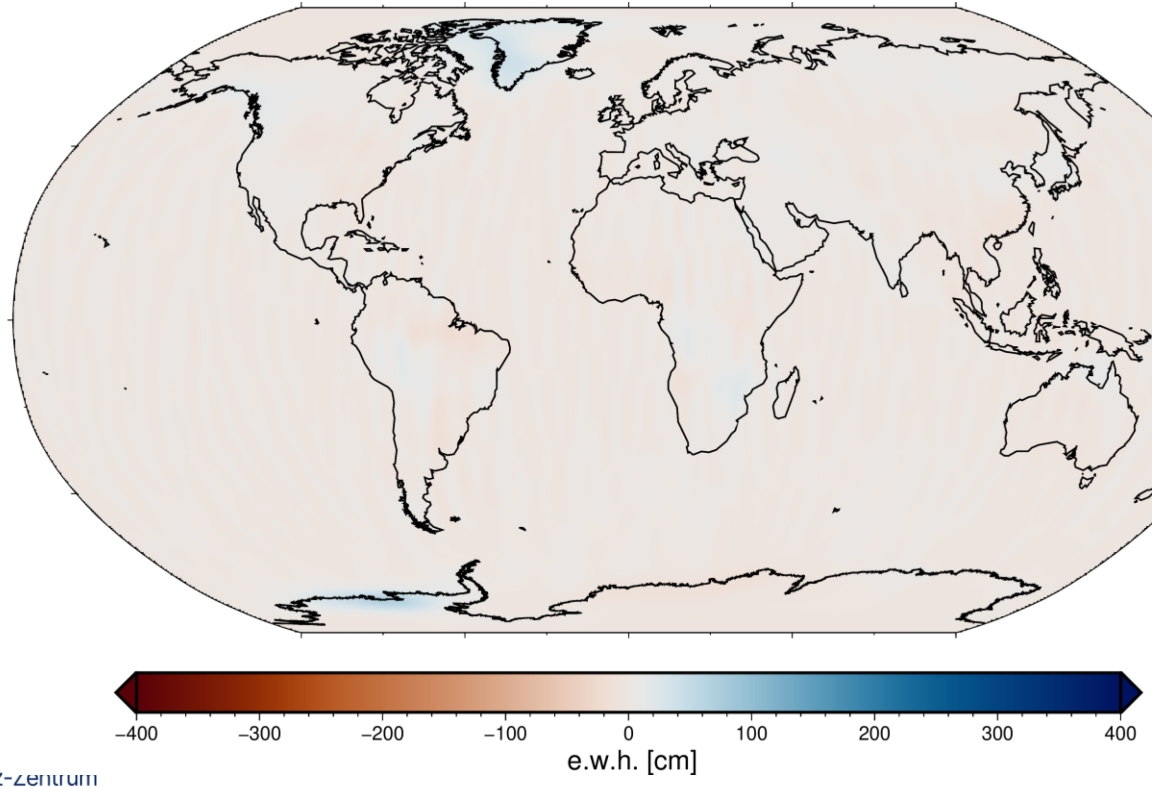
Gaussian filter
400km



GFZ Helmholtz-Zentrum für Geoforschung

Filtering

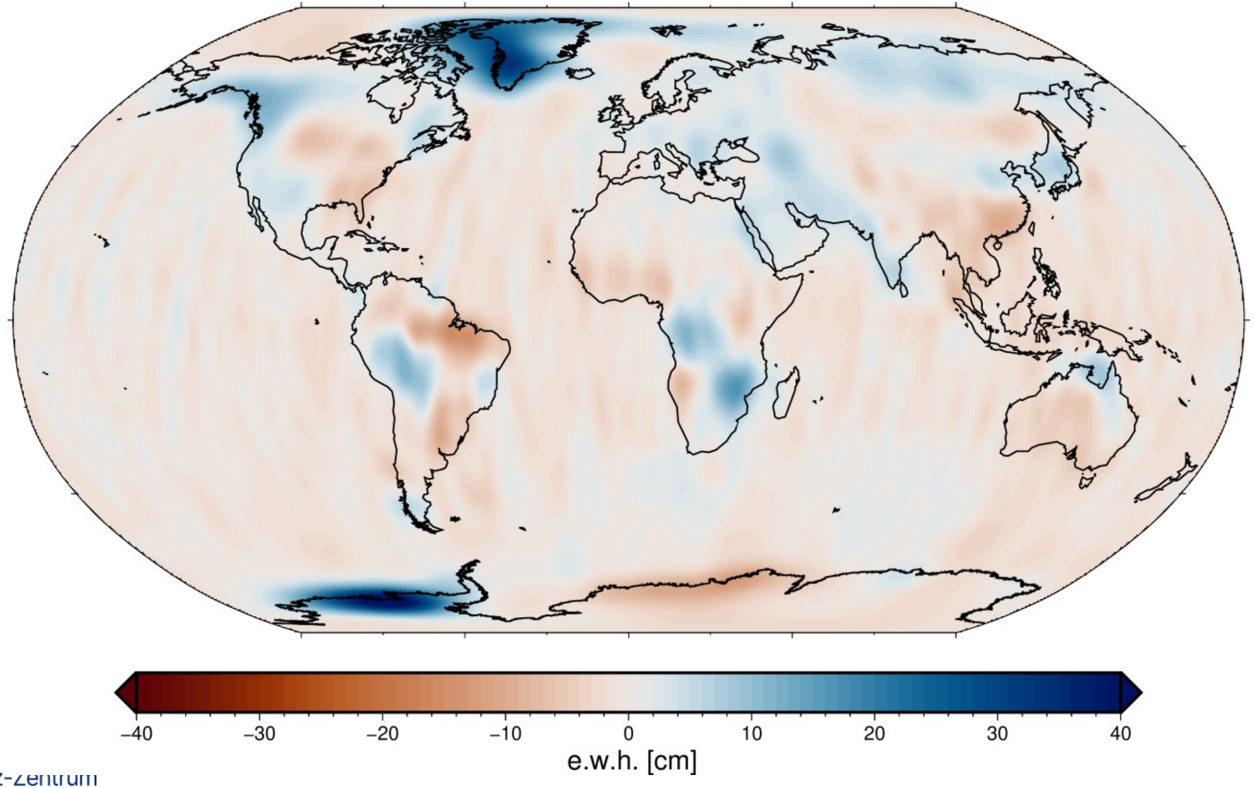
Gaussian filter
400km



GFZ Helmholtz-Zentrum für Geoforschung

Filtering

Gaussian filter
400km



Gaussian Filter

- The filtered value of each grid point is the weighted sum of all other grid points
- The filter depends only on the filter radius r

Spatial filtering:

$$W(\alpha) = \frac{b}{2\pi} \frac{\exp(-b(1-\cos(\alpha)))}{1-\exp(-2b)}$$

with $$b = \frac{ln(2)}{1-\cos\left(\frac{r}{R}\right)}$$

α = central angle

r = filter radius

Gaussian Filter

- Spatially filtering computationally expensive
- Gaussian filter can also be understood as down-weighting of higher degree spherical harmonic coefficients

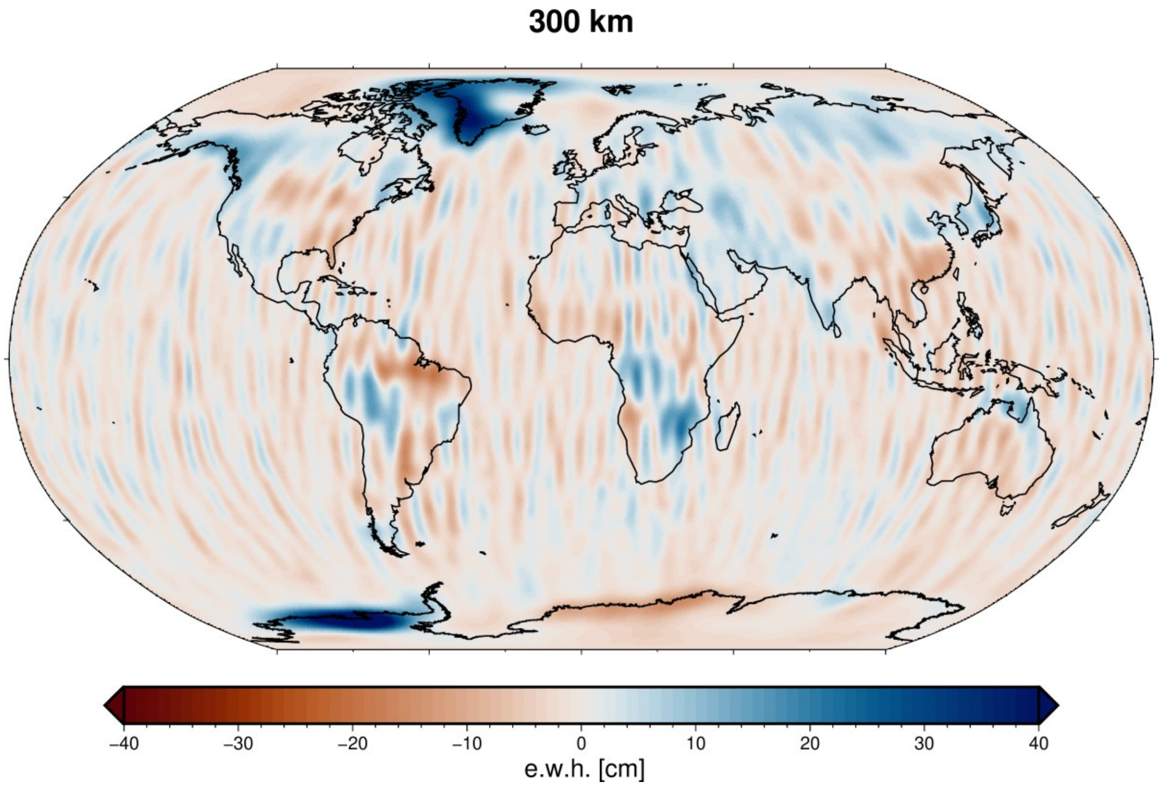
- Recursive formula:

$$W_0 = 1 \qquad W_1 = \frac{1+\exp(-2b)}{1-\exp(-2b)} - \frac{1}{b} \qquad W_{n+1} = -\frac{2n+1}{b} W_n + W_{n-1}$$

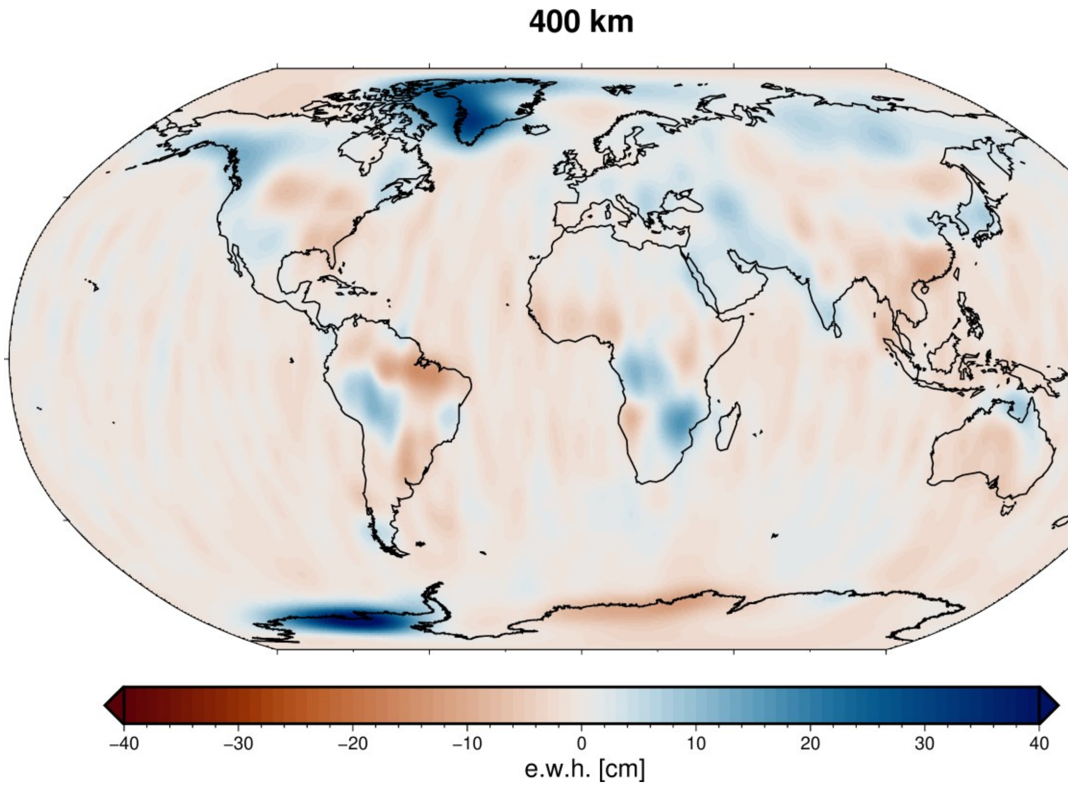
- Filtered coefficients: $\widehat{a_{nm}} = W_n a_{nm}$

GFZ Helmholtz-Zentrum für Geoforschung

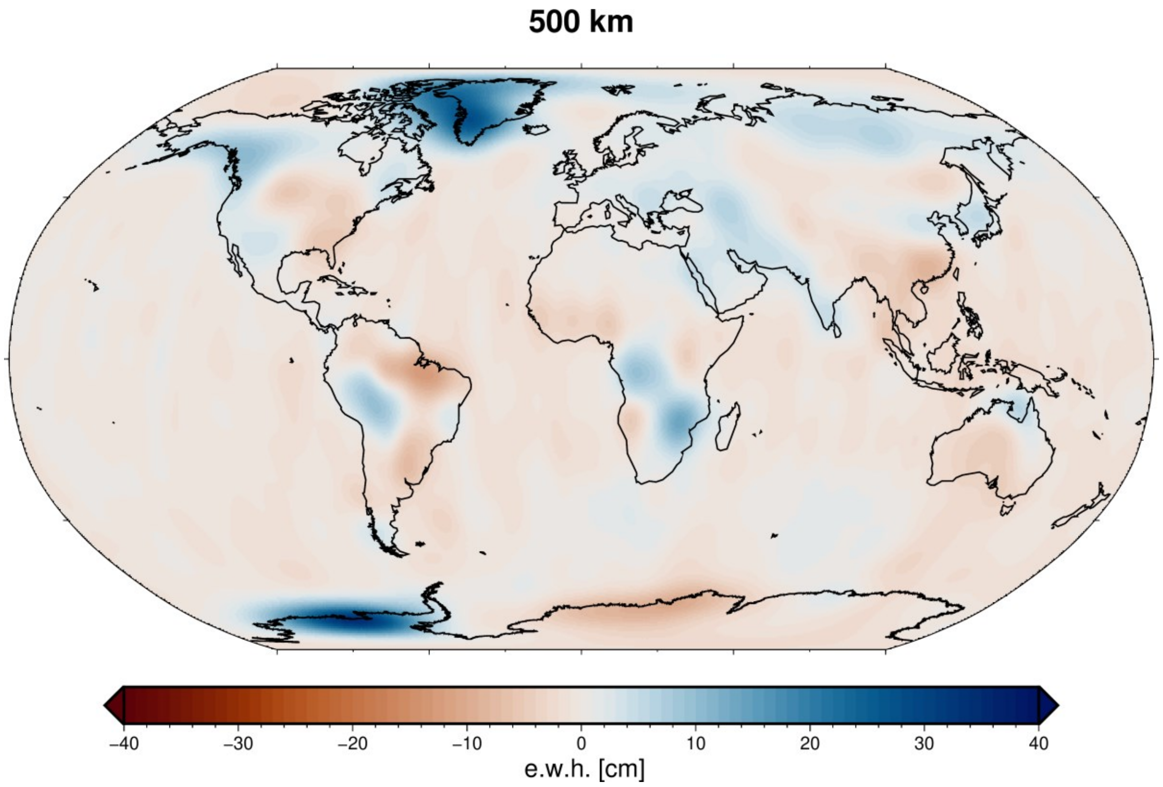
Gaussian Filter



Gaussian Filter

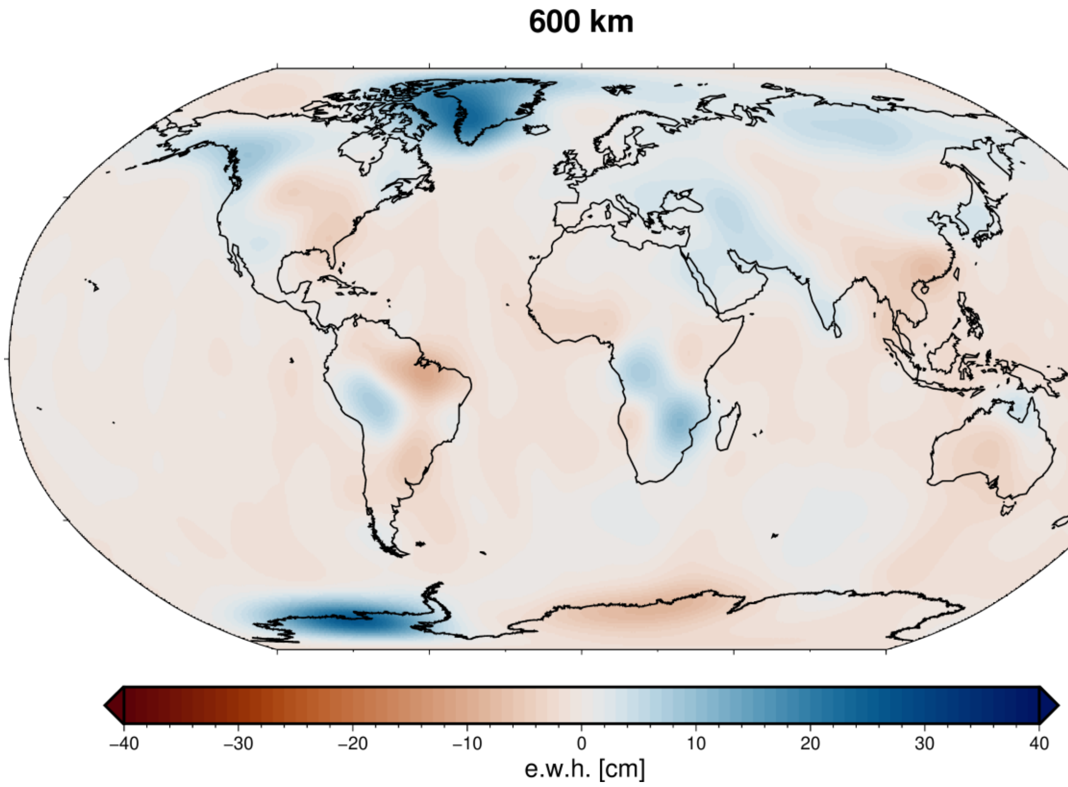


Gaussian Filter



Helmholtz-Zentrum
für Geoforschung

Gaussian Filter



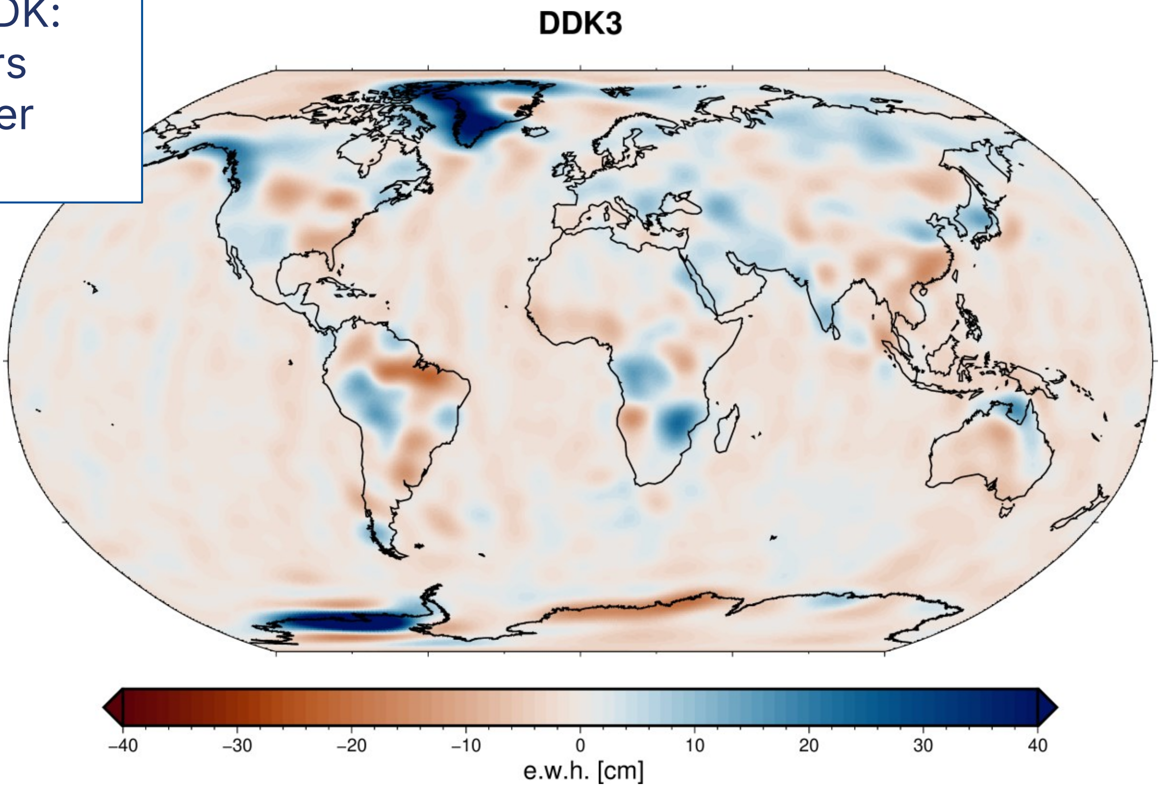
GFZ Helmholtz-Zentrum für Geoforschung

Anisotropic Filter

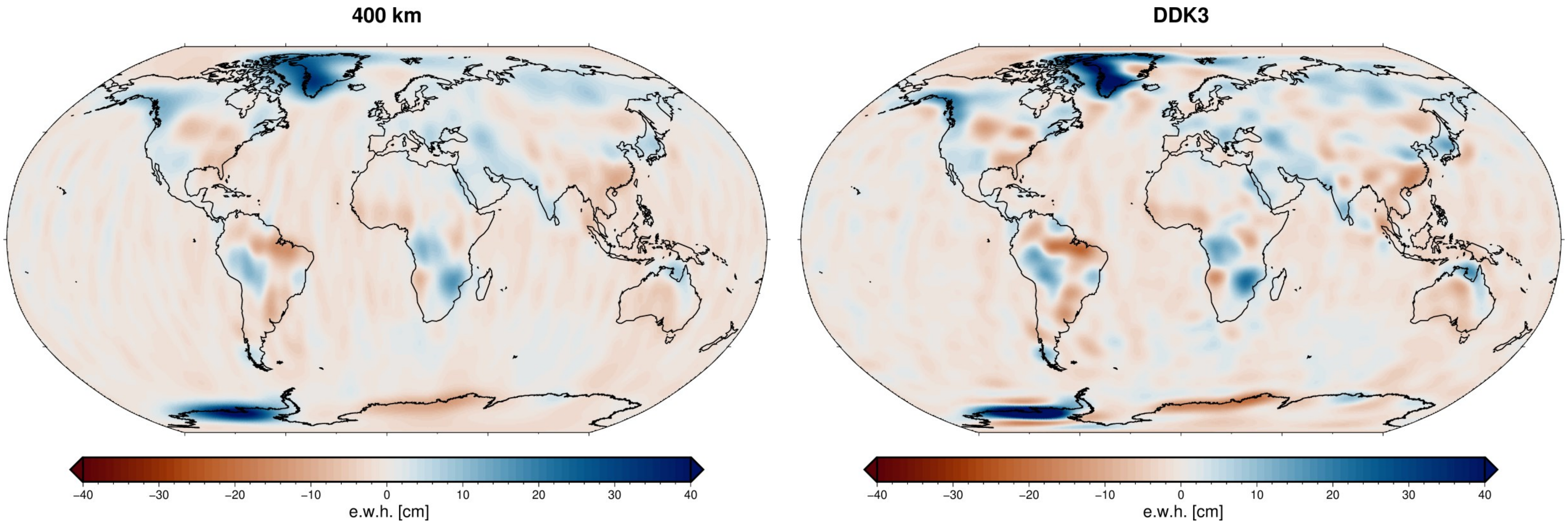
- Gaussian Filter does not consider the anisotropic structure of the spatially correlated striping errors
- Several special filters have been developed:
  - DDK filter
  - Swenson-Wahr destriping filter
  - VDK filter (time-variable DDK filter)
  - ...

Example DDK3

For DDK and VDK: smaller numbers indicate stronger filter!



DDK vs Gaussian Filter



GFZ Helmholtz-Zentrum für Geoforschung

Anisotropic Filter

- Pro:
  - Specifically designed for spatially correlated striping errors of GRACE and GRACE-FO
  - Better preserve signal location and amplitude while reducing striping errors
- Contra:
  - More challenging to implement
  - Introduces additional a-priori information
  - Filtering needs to be done in the spherical harmonic domain

https://icgem.gfz-potsdam.de provides DDK pre-filtered (DDK1 to DDK8) SH coefficients for SDS solutions (GFZ, JPL, CSR)!

3. Corrections

Corrections

Besides filtering, other corrections need to be applied between Level-2 and Level-3 data

- Replacement of low-degree harmonics
- Correction or removal of solid Earth signals
  - GIA
  - Earthquakes
- Correction of S2 tidal aliasing errors

Replacement of Low-Degree Harmonics

- c_{20}, flattening of the Earth, poorly determined by GRACE/GRACE-FO
- c_{30} poorly determined, too, at the end of life of GRACE
- Can be replaced by estimations of these coefficients from SLR or GRACE+SLR data

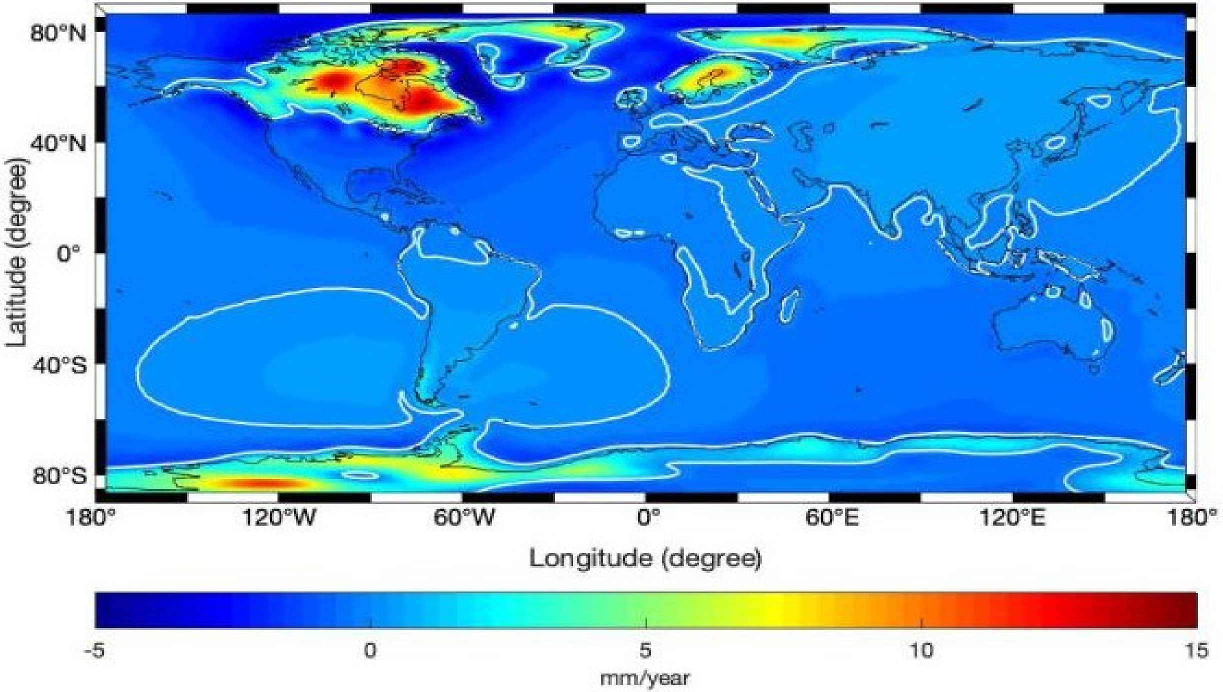
- c_{10}, c_{11}, and s_{11} describe geocentre variations (distance centre of mass and centre of figure); GRACE/GRACE-FO is insensitive
- Replaced by an external source or approximated from monthly GRACE/GRACE-FO SH coefficients

Solid Earth Signals

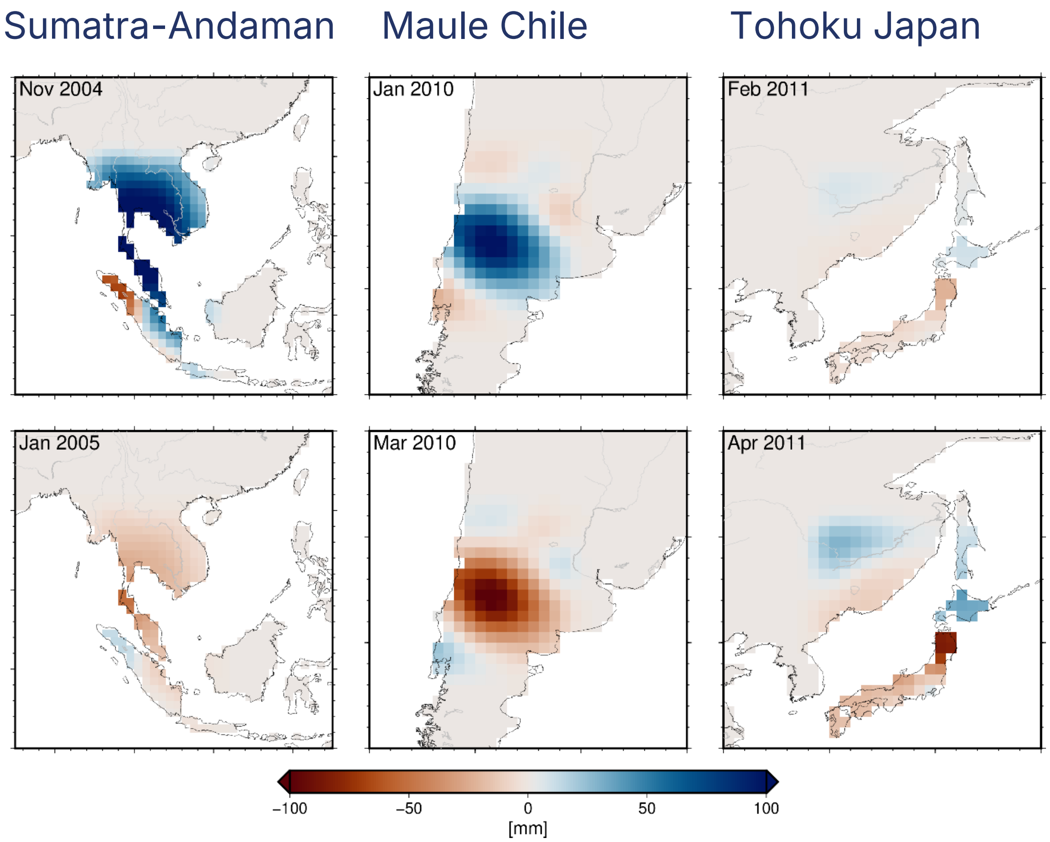
- Not only water (liquid or solid) is causing mass redistributions, but also solid Earth phenomena:
  - Glacial isostatic adjustment (GIA)
  - Co- and post-seismic deformations of earthquakes
- For separation between hydrology and solid Earth signals, one of the two has to be modelled to be removed from the GRACE/GRACE-FO observations

GIA

ICE-6G_D


80°N
40°N
0°
40°S
80°S
Latitude (degree)
180°
120°W
60°W
0°
60°E
120°E
180°
Longitude (degree)
-5
0
5
10
15
mm/year

Earthquake Signals

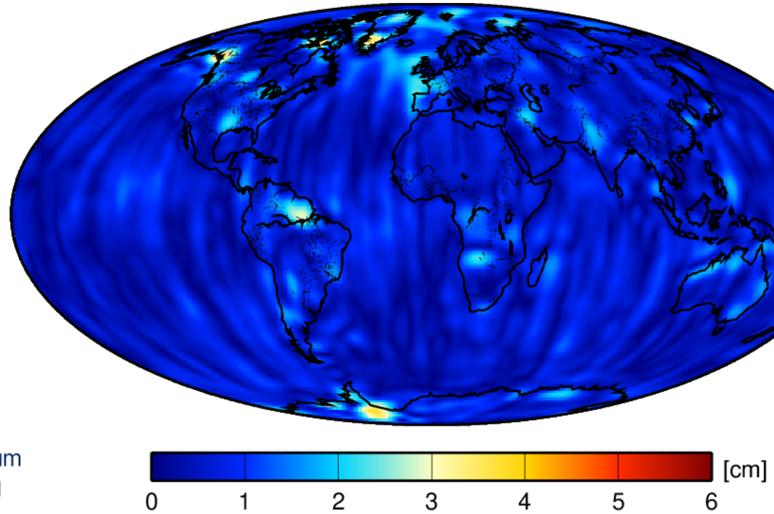


Correction of S2 tidal aliasing errors

- Removing ocean tidal signals from satellite observations is crucial during GRACE processing (lecture by M. Hart-Davis and L. Shihora)
- Temporal aliasing of unmodelled/insufficiently modelled tide signals remains
- Significant aliasing frequency with a period of 161 days, which relates to the semi-diurnal solar tide S2 present in the ocean and atmosphere

Correction of S2 tidal aliasing errors

- To mitigate, bias, linear trend, annual, semi-annual, and 161 days periodic signals (separate for GRACE and GRACE-FO) are fitted to the time series of monthly SH coefficients
- Then the 161 days periodic signal is subtracted.



Absolute amplitude of the estimated S2 tidal alias signal for the VDK3 filtered GFZ RL06 Level-2B products

Example of a Processing Chain I

Input: Stokes coefficients (Level-2)
Processing:

- Reduce mean (2002/04 - 2020/03)
- Filtering with VDK (Horvath et al., 2018)
- Replace coefficients C_{20}
- Subtract model of GIA ICE-6G_D (VM5a)
- Insert coefficients of geocentre motion C_{10}, S_{11}, S_{11}
- Subtract aliasing signals of S2 ocean tide (161d period)

Output: Stokes coefficients (Level-2B)

Example of a Processing Chain II

Input: Stokes coefficients (Level-2B)

- SH synthesis of L2B data to 1° global grid, ellipsoidal reference
- Remove empirically estimated co-seismic signals of mega-thrust earthquakes (Sumatra-Andaman 2004, Chile 2010, Tohoku 2011)
- (Mask out oceans, Greenland, and Antarctica)

Output: Monthly 1° TWS grids

Shortcut to Level-3 data

- Gravity Information Service (gravis.gfz.de) provides ready to use Level-3 data
- Interactive data viewer
- Also L2B data can be downloaded
- Data sets divided into land, ocean, and ice domain

Wrap-Up Level-2 to Level-3 data processing

- GRACE/GRACE-Fo Level-2 data can be converted to gridded data with the spherical harmonic synthesis
- Filtering necessary to mitigate anisotropic spatially correlated errors
- Replacement of low-degree harmonics from other data sources necessary
- Solid Earth signals should be corrected for
- Ready-to-use data is available at GravIS (gravis.gfz.de)

Are we done?

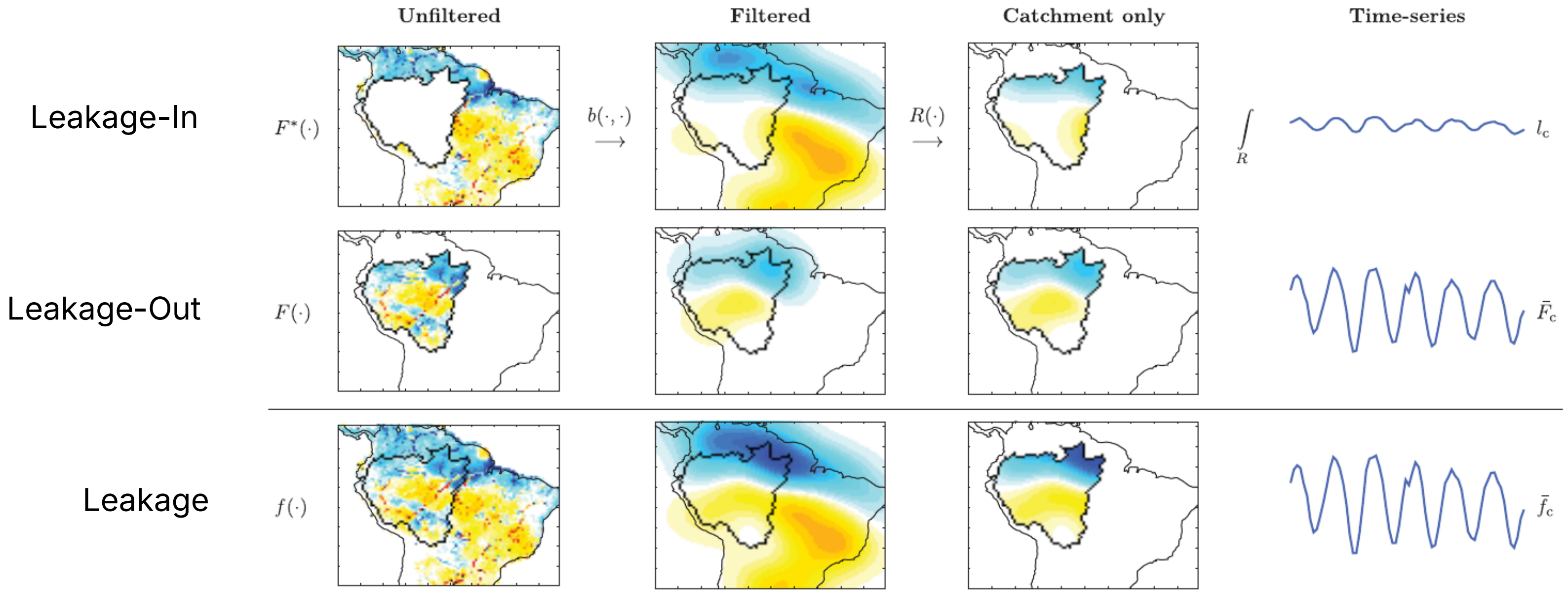
Not yet!

4. Leakage

Leakage

- GRACE/GRACE-FO data band-limited resolution and coarse spatial scale
- Leakage describes the inability to localise signals in the GRACE-derived data sets exactly
- One cause of leakage is signal attenuation due to the distance between the signal source on the Earth's surface and the observing satellites.
- Another cause is the processing of Level-3 data, foremost filtering
- Together, leakage leads to apparent signal loss (leakage out) or gain (leakage in) inside a given integration region

Leakage



Courtesy to Brahma Vishwakarma et al. (2016)

Leakage Mitigation

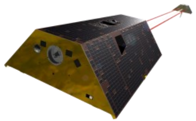
- Over the last 20 years, a large number of methods have been published to mitigate leakage
- One of them is implemented in the GravIS data sets
- Others are, for example, based on hydrological models to assess the leakage effect or forward-modelling approaches, where the geometry of surface mass change is assumed a priori

5. Mascons

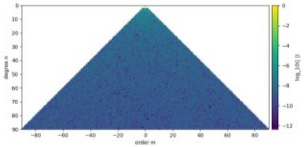
Mascons

Spherical harmonics approach

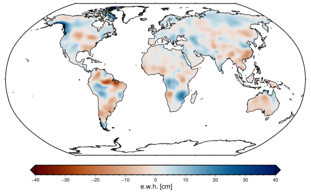
L1 data → L2 data → L3 data



unconstrained



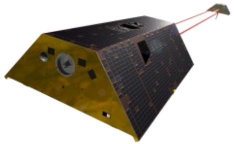
filtering



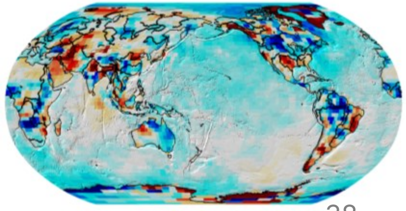
Both use some a priori information!

Mascons approach

L1 data → L3 data



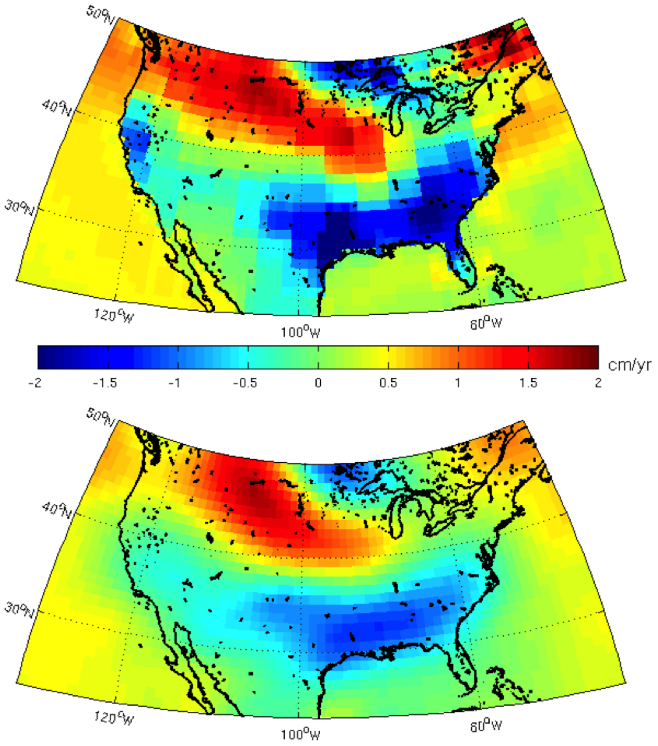
Constrained by prio information



Differences between Mascons and SH-based Level-3 data

JPL Mascons

SH-based



Operational mascons:

- JPL
- CSR
- GSFC
- (ANU)

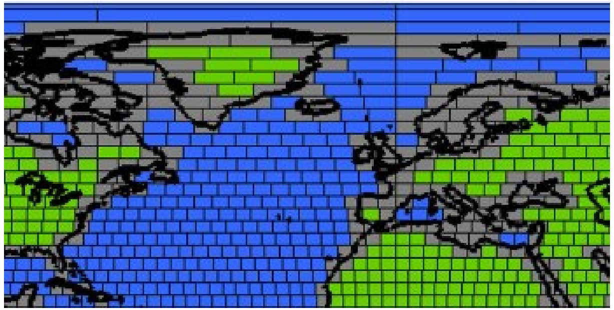
Source: https://podaac.jpl.nasa.gov/highlights/JPL_GRACE_Mascon_products_2015_1117

Remarks on Mascons

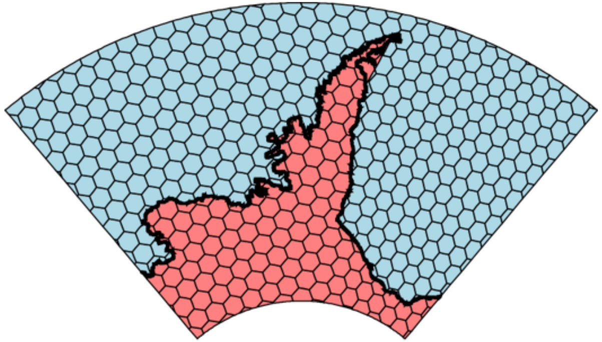
- Mascons are computed on non-lon-lat grids and afterwards down sampled to regular lon-lat grids
- All mascons available have their native grid

Remarks on Mascons

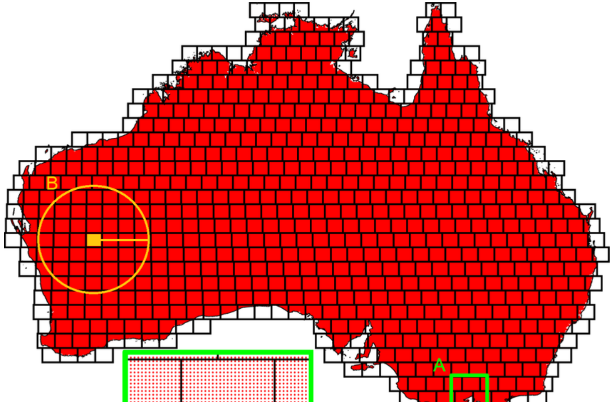
- Mascons are computed on non-lon-lat grids and afterwards down sampled to regular lon-lat grids
- All mascons available have their native grid



JPL:
3° spherical caps on a grid with equal latitude spacing



CSR:
1° spherical caps on a geodesic grid



GSFC
1° equal area cells on a grid with equal latitude spacing

Remarks on Mascons

- Mascons are subject to less leakage
- Grid-wise comparison between different mascons data sets and sh-based data sets is difficult
- 0.25° grid for mascons vs 1° grid for SH-based L3 data, does not mean that the mascons have a higher spatial resolution

Any questions?