



NERO GRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

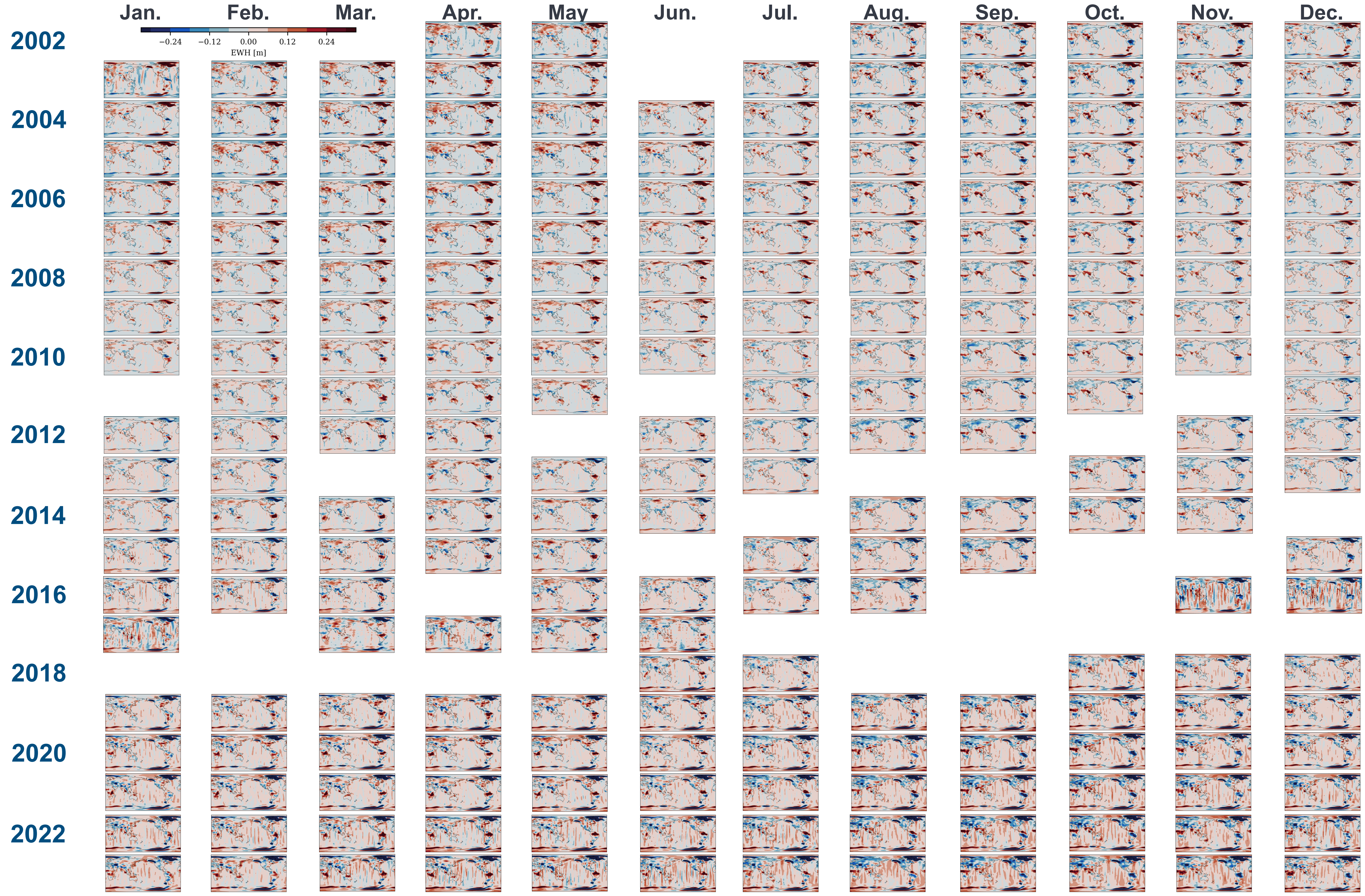
International Spring School
Neustadt an der Weinstraße, Germany, March 10-14, 2025

Background Modelling for Satellite Gravimetry

Linus Shihora (GFZ Helmholtz Centre for Geosciences)



GRACE Monthly Solutions



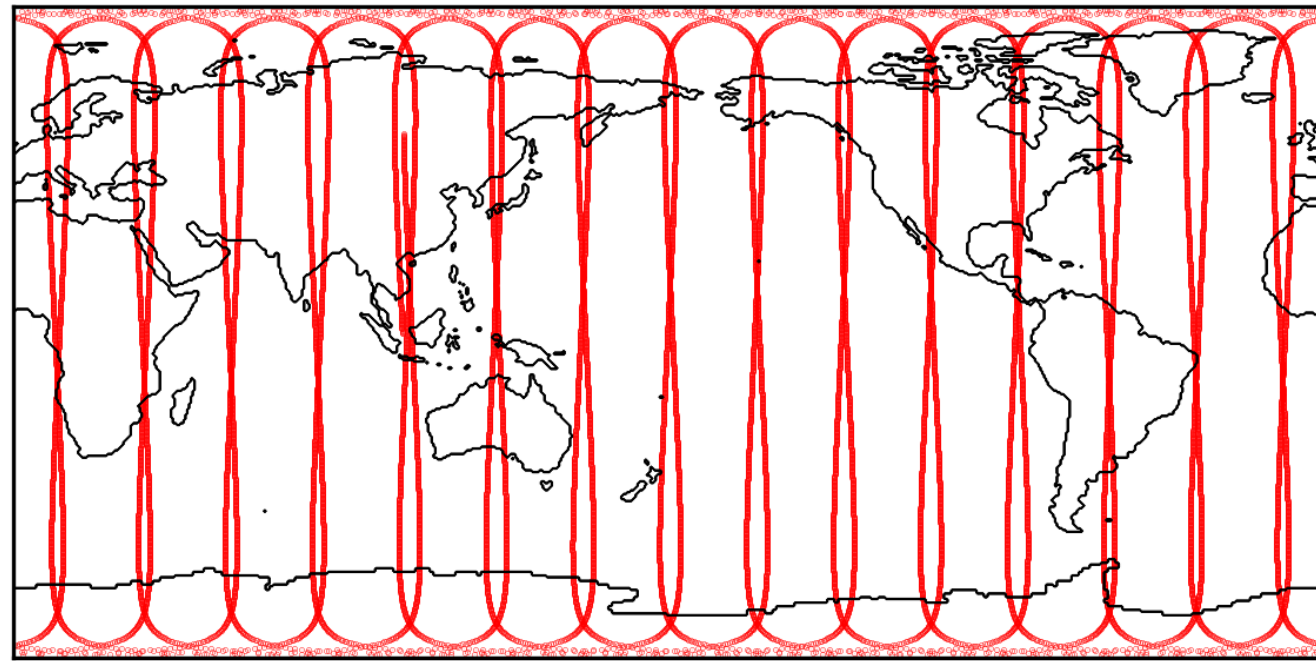
GRACE Groundtrack Coverage

Main Objective: Measuring global mass redistributions

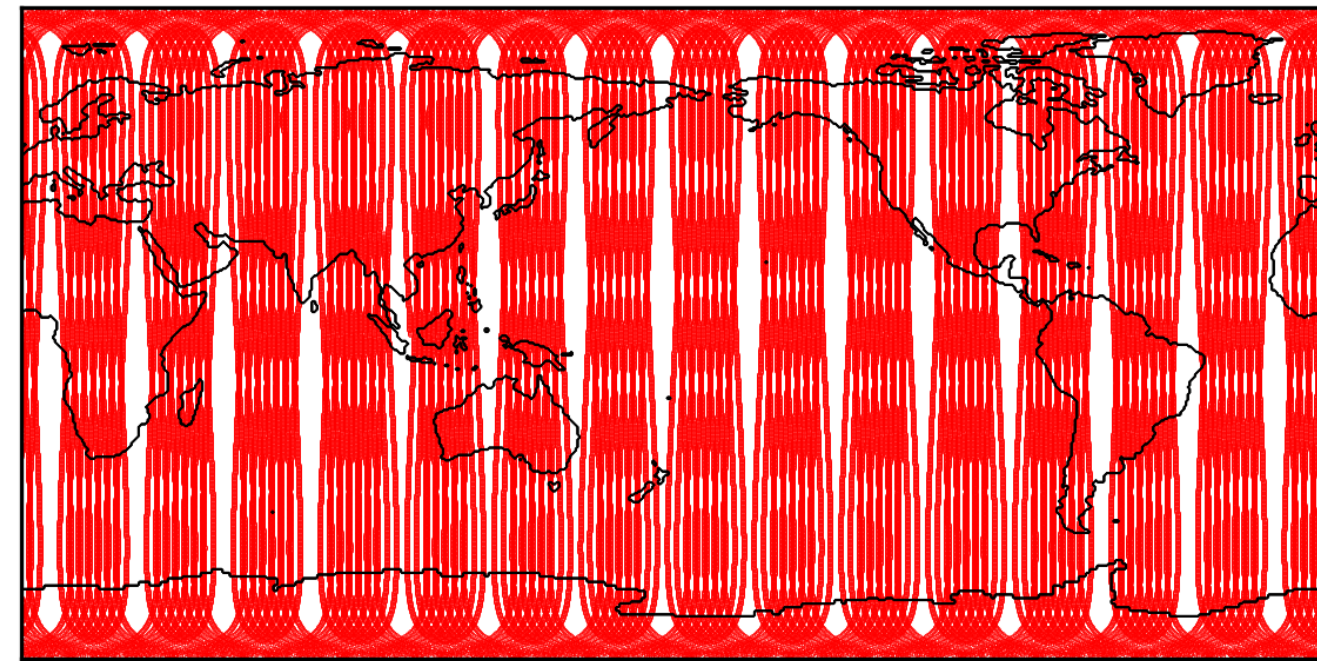
- Data is usually accumulated for 1 month to compute global gravity solution

Example groundtrack after:

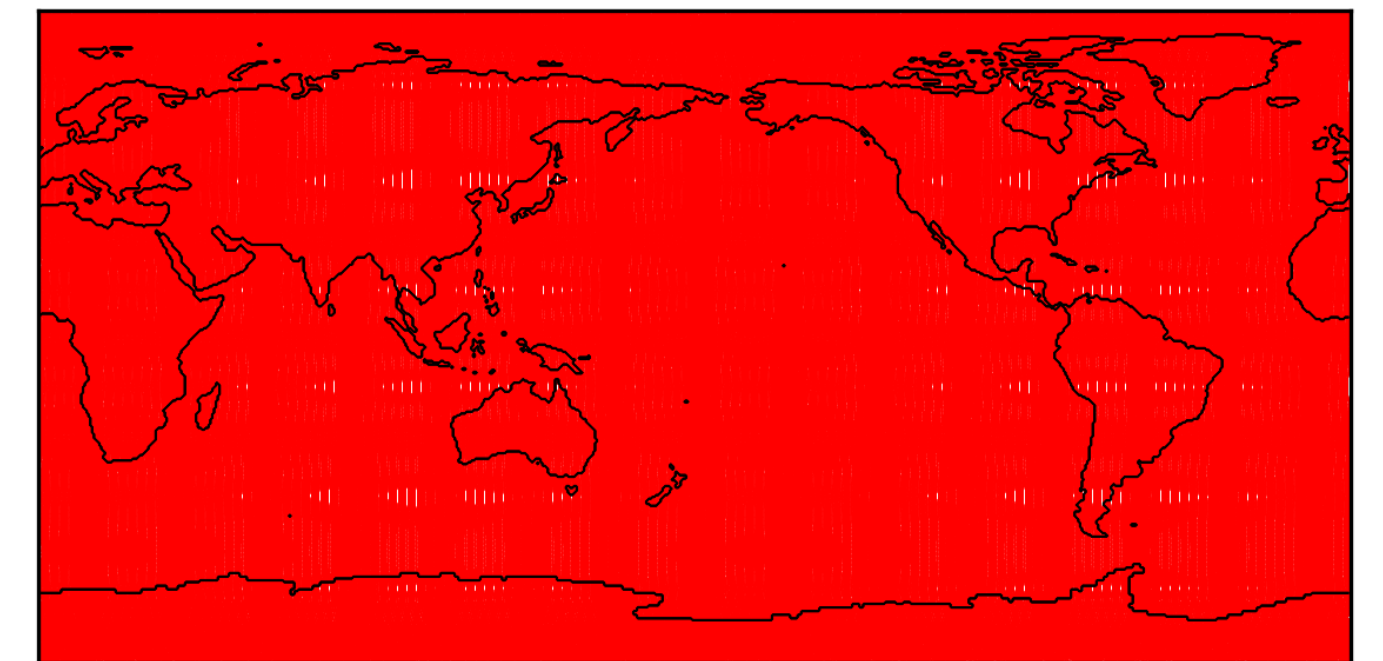
1 day



1 week



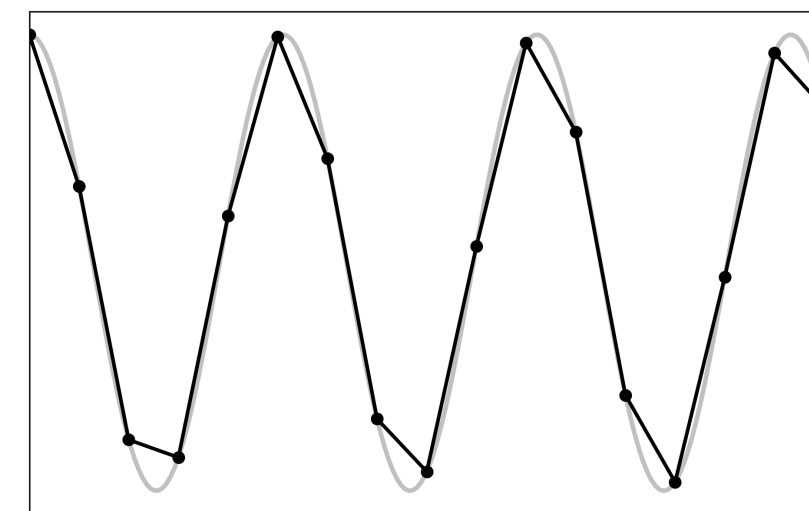
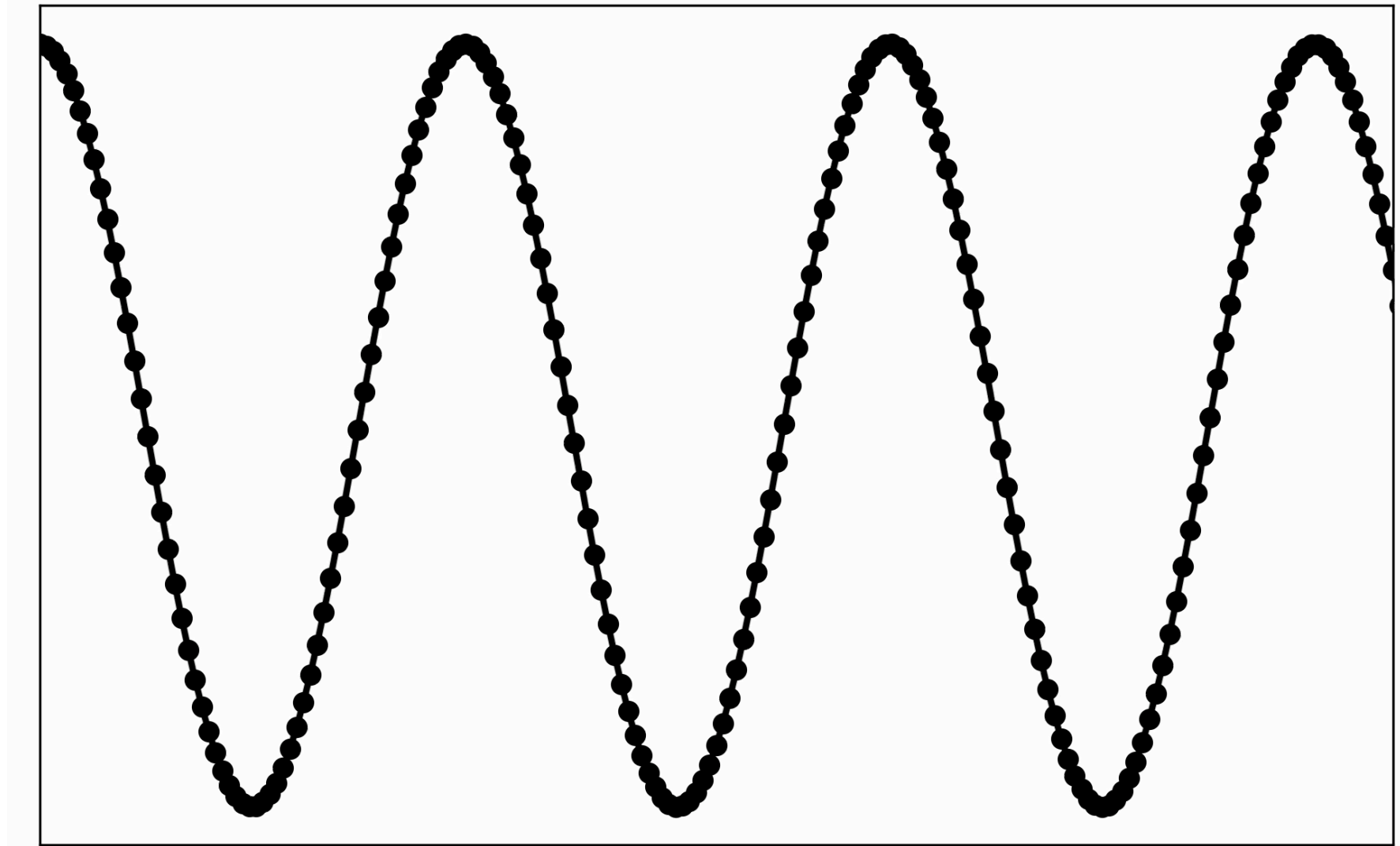
1 month



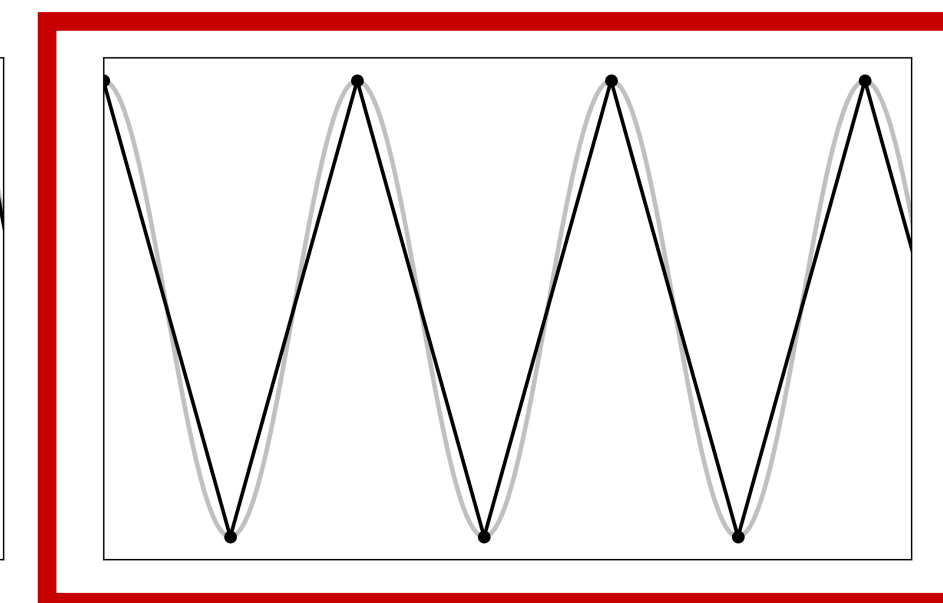
Temporal Aliasing

- Grace orbital period: ~ 90 min
- For final gravity solutions data is accumulated for ~ 30 days
- Signals with periods below twice the accumulation period can induce spurious signals
- Even with along-track data high frequency signals can not be resolved
- **Example:** sampling of sine-wave

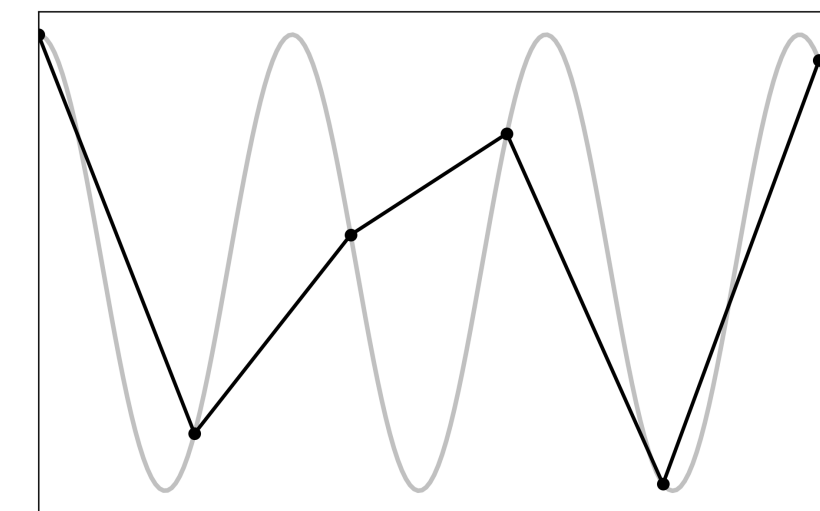
Example: Sampling a Sine Wave



resolvable



$$\Delta t = T/2$$

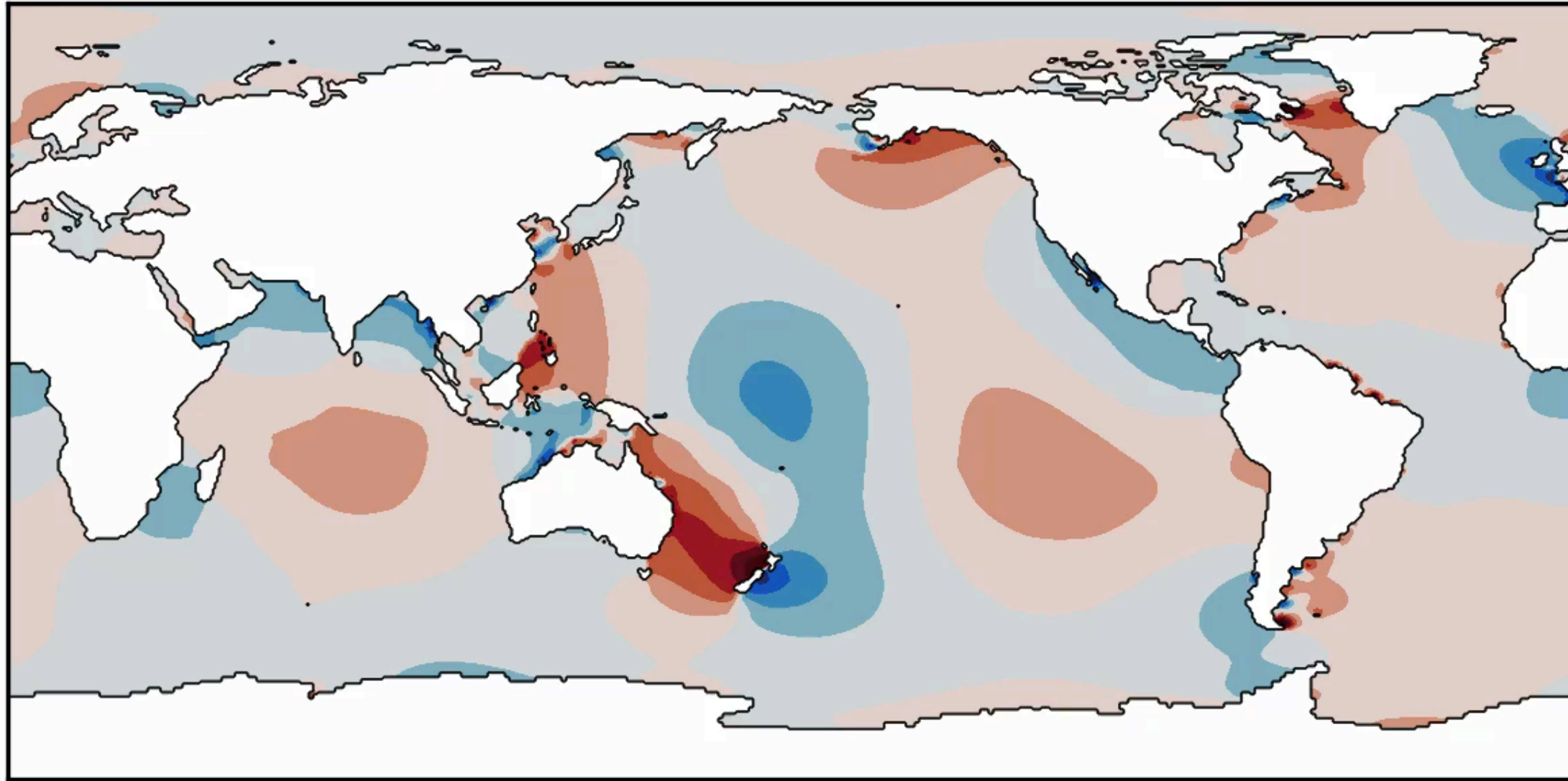


not resolvable

Temporal Aliasing

Example: M2 Tide & GRACE Groundtracks

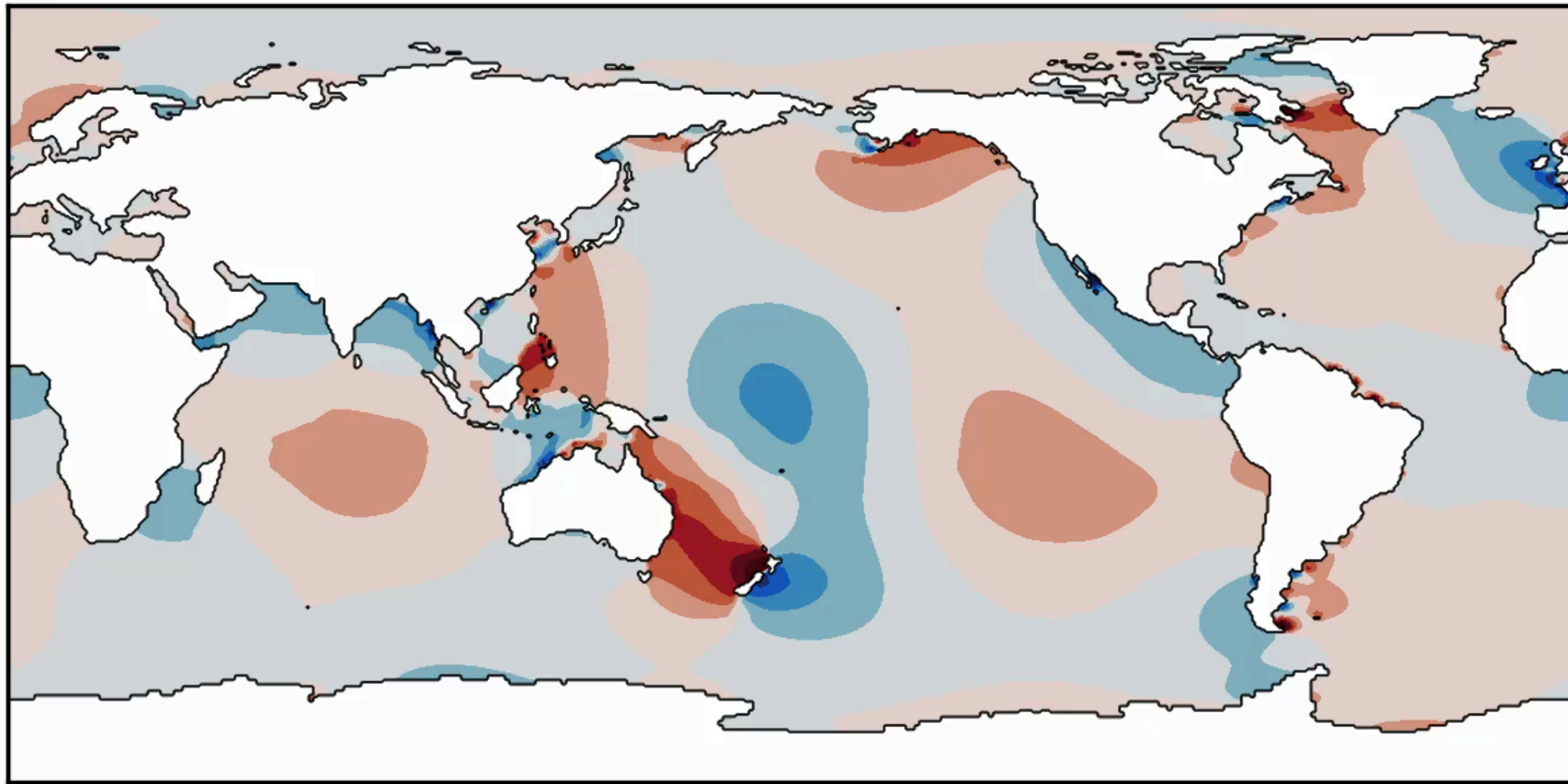
M2 Tidal Signal



Temporal Aliasing

Example: M2 Tide & GRACE Groundtracks

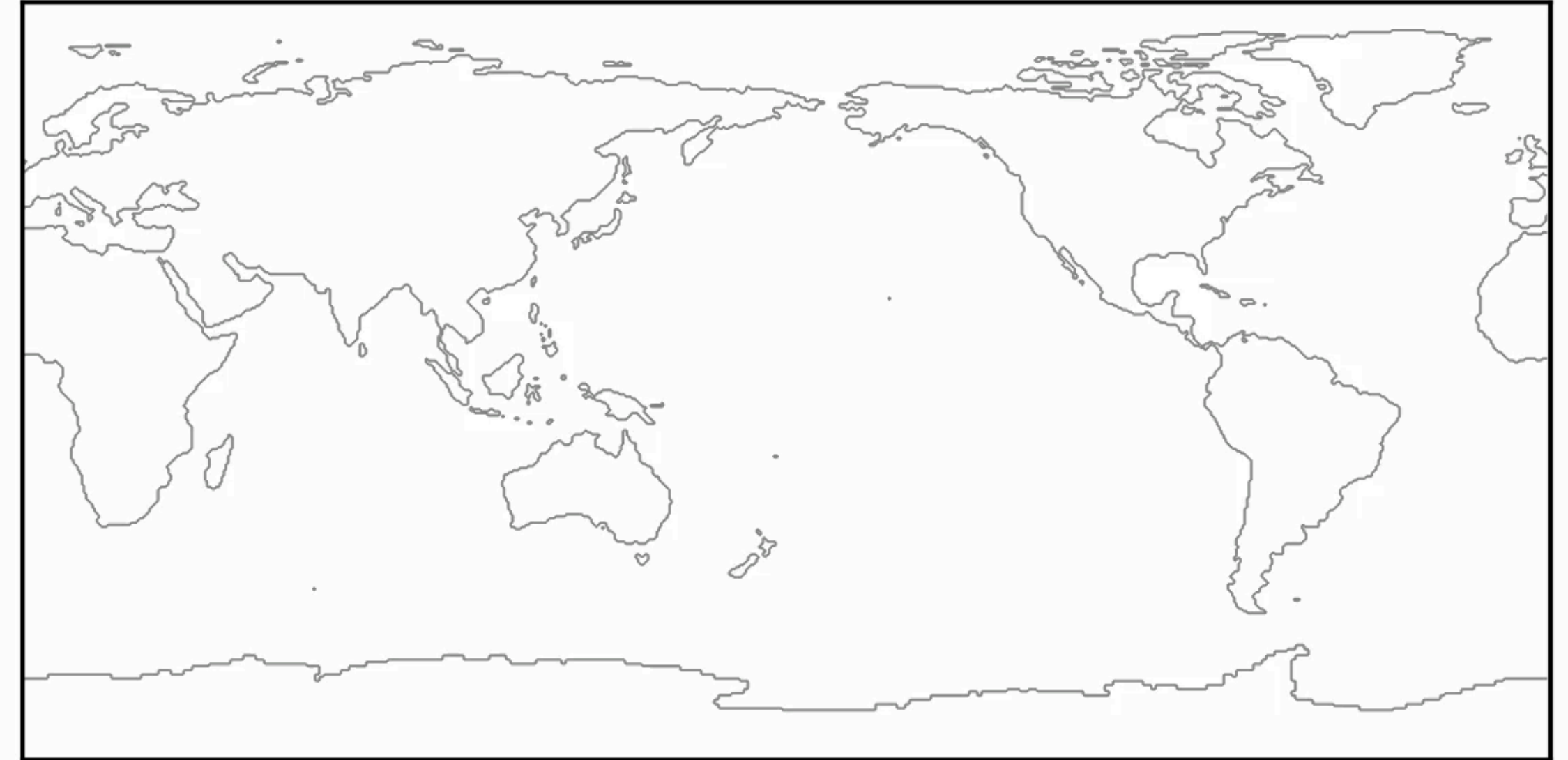
M2 Tidal Signal



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GRACE-FO Groundtrack

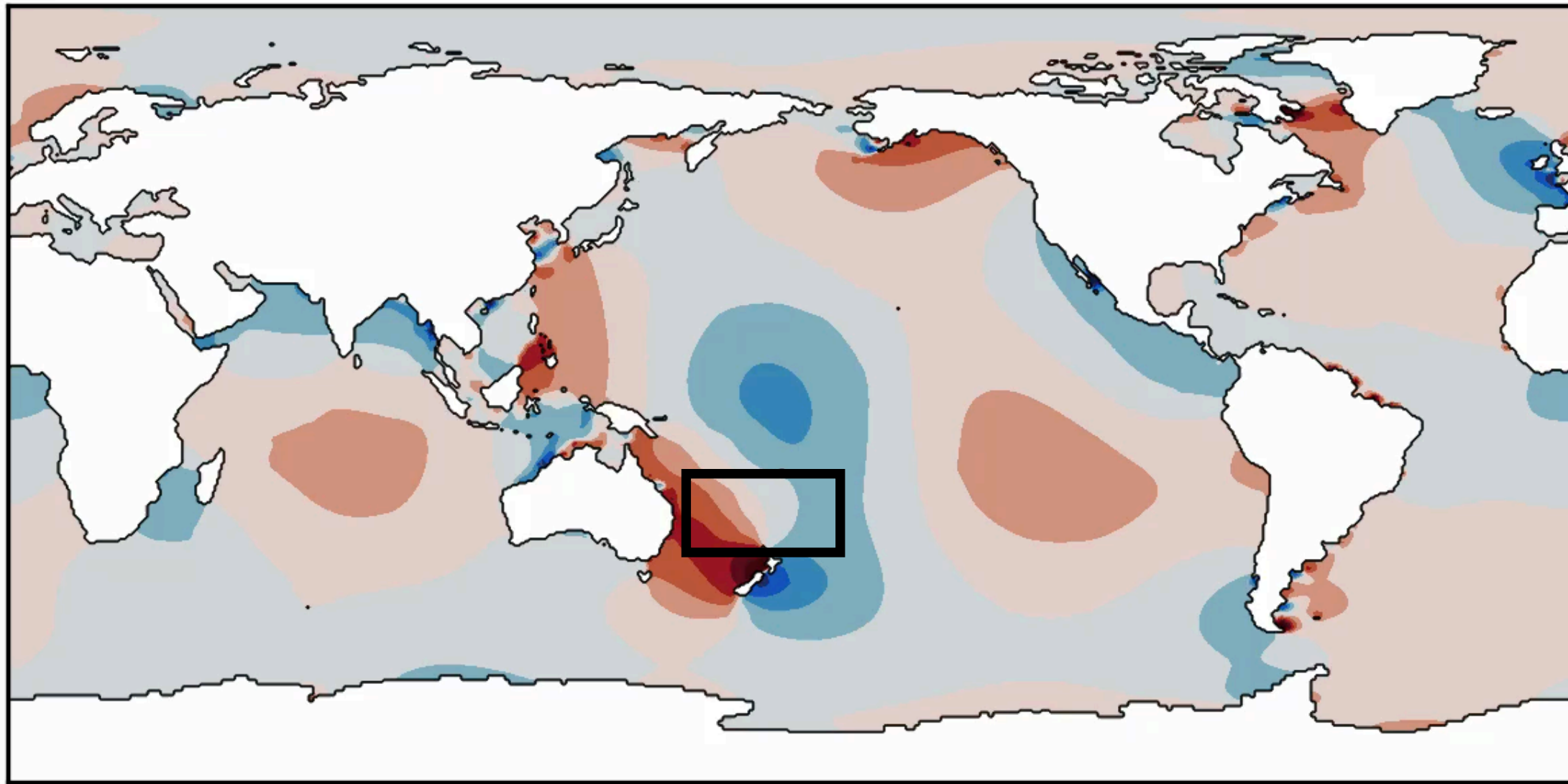
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Temporal Aliasing

Example: M2 Tide & GRACE Groundtracks

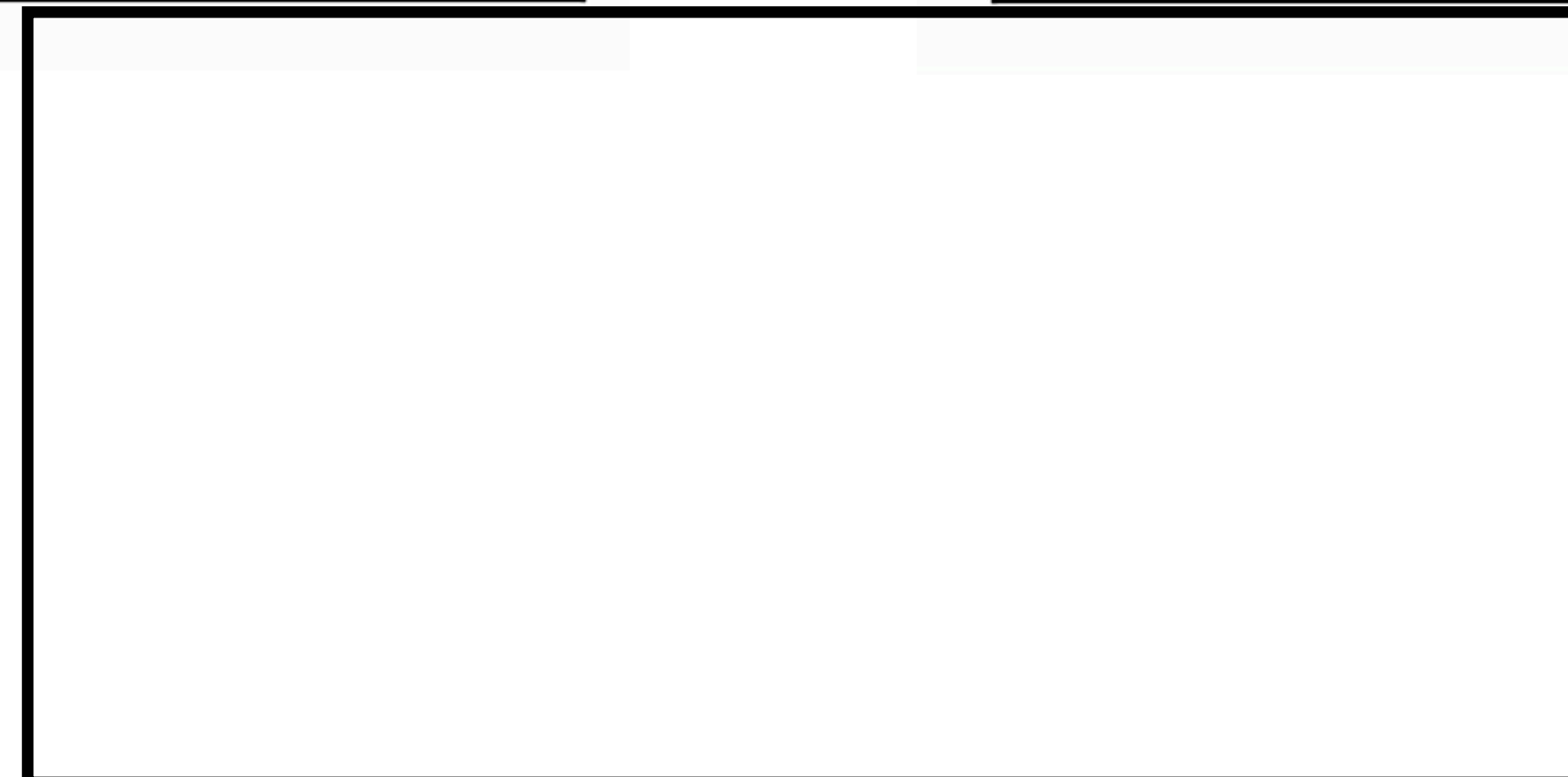
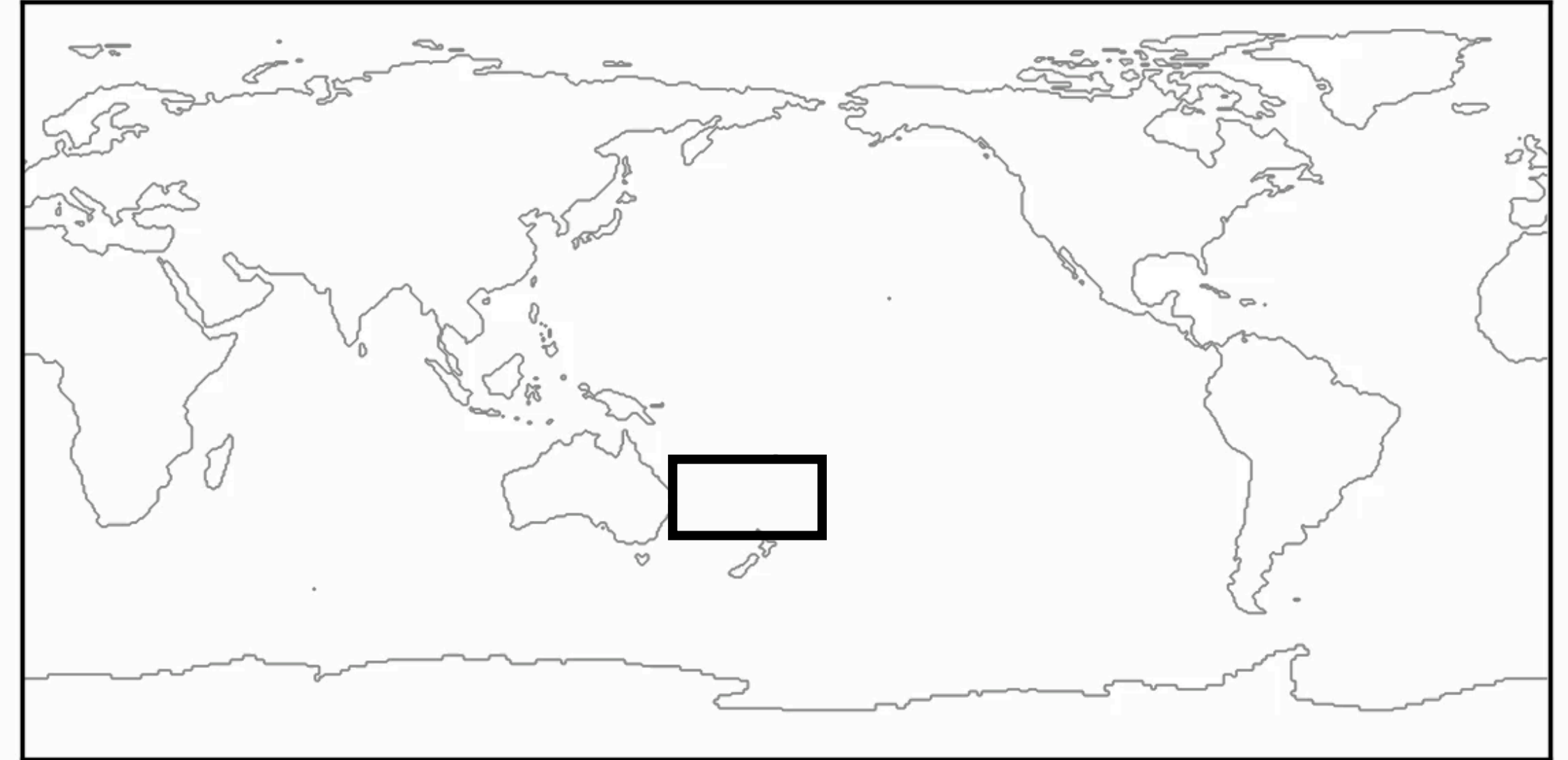
M2 Tidal Signal



+

GRACE-FO Groundtrack

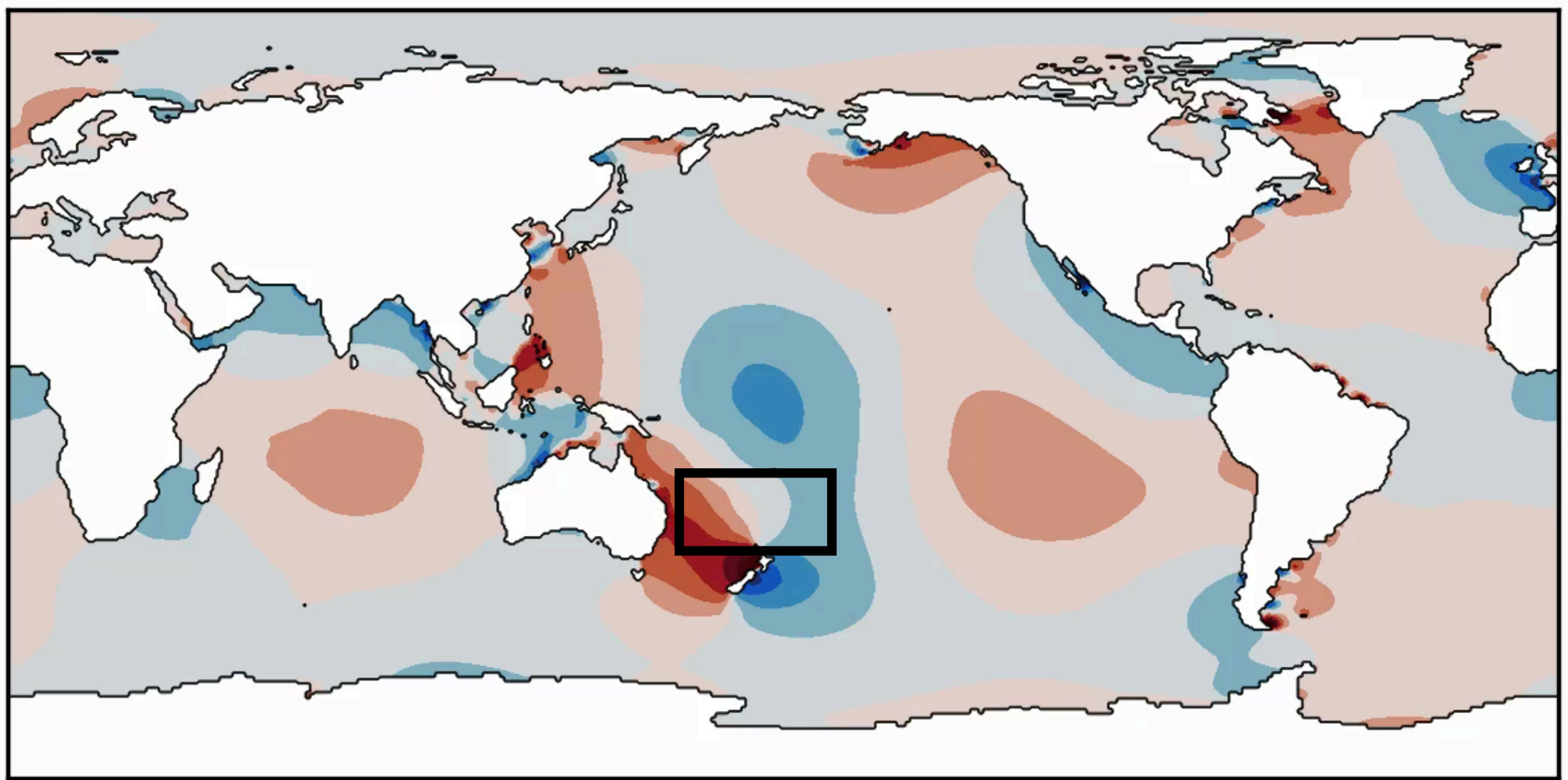
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Temporal Aliasing

Example: M2 Tide & GRACE Groundtracks

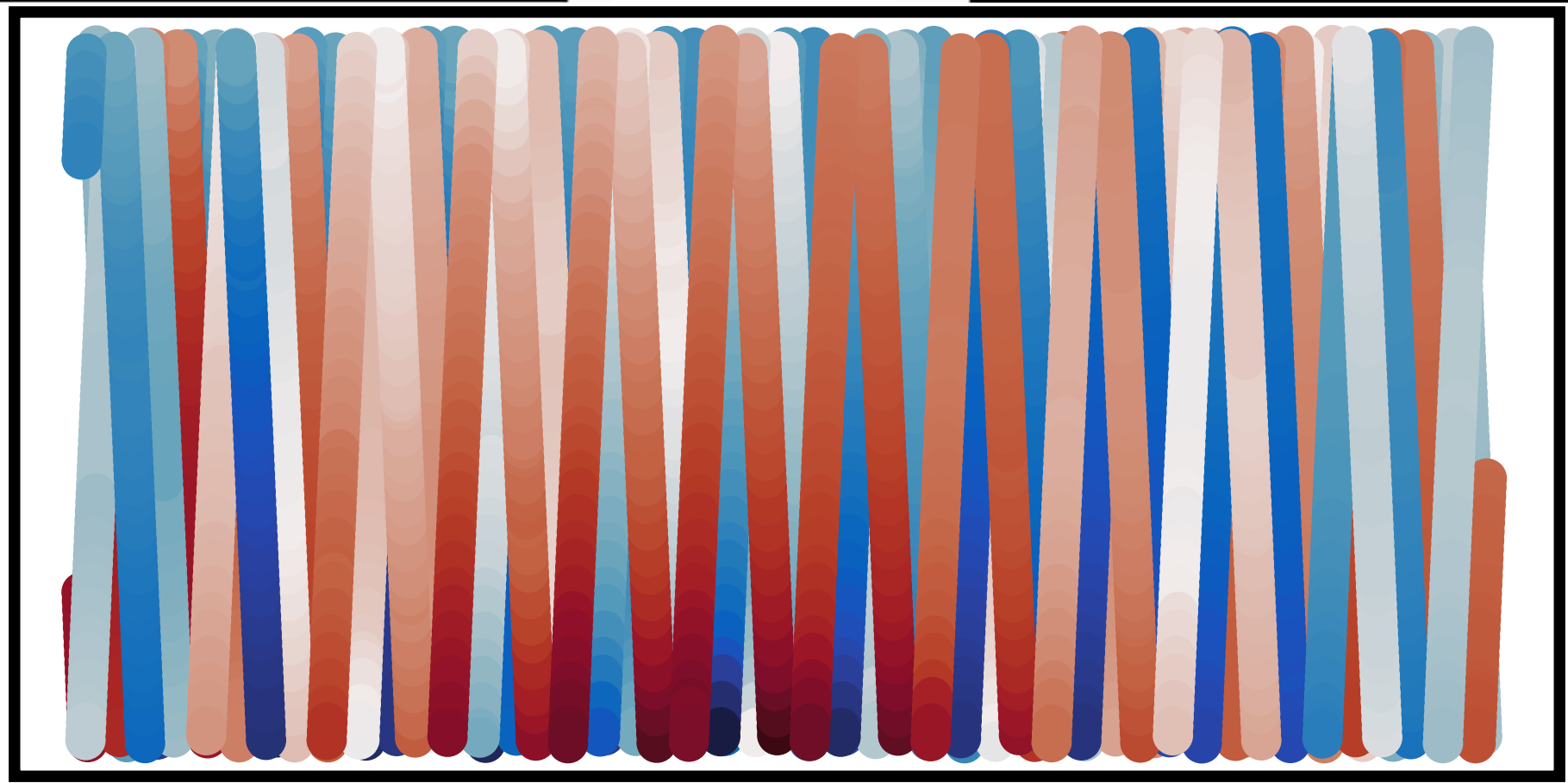
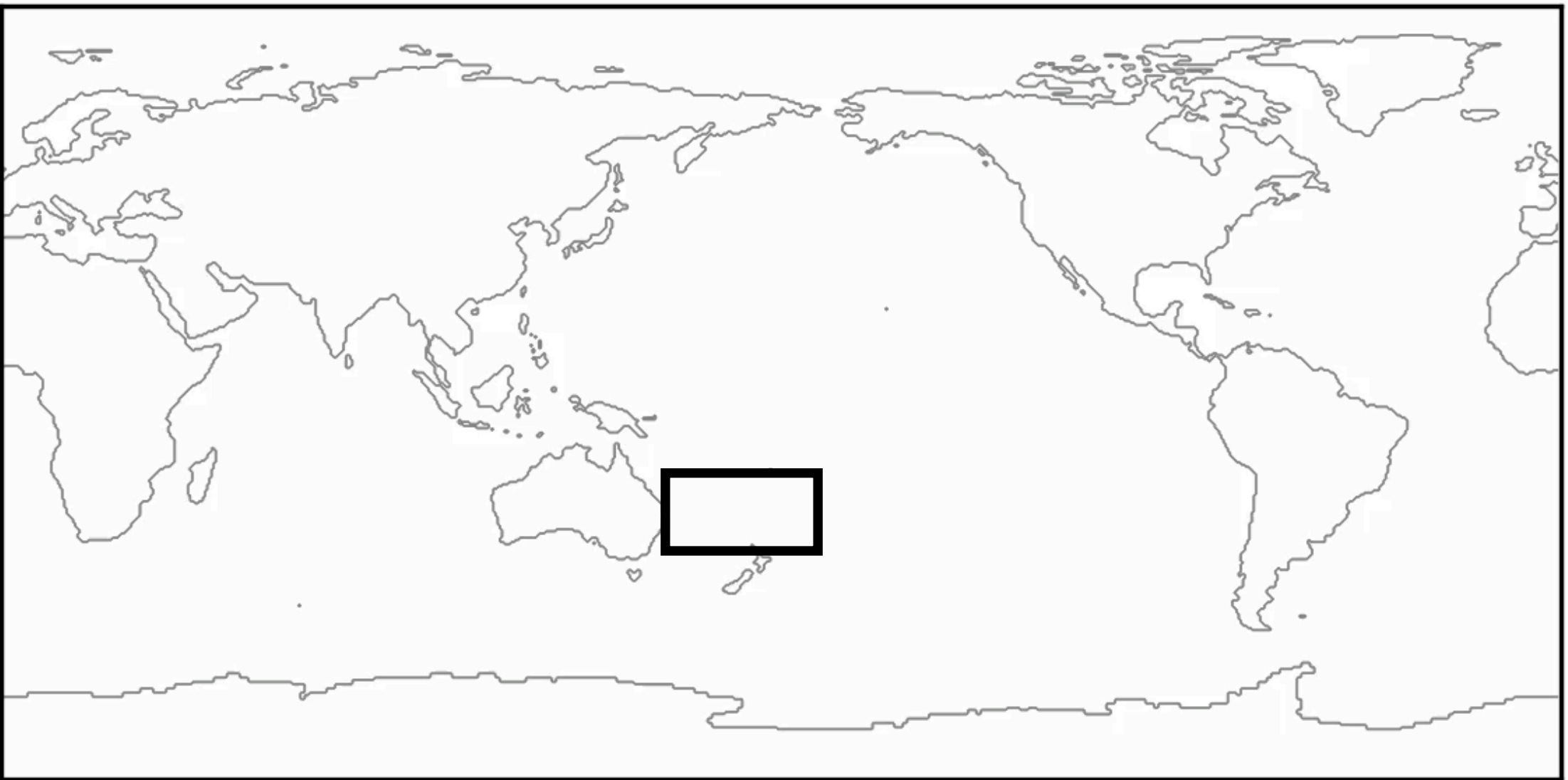
M2 Tidal Signal



+

GRACE-FO Groundtrack

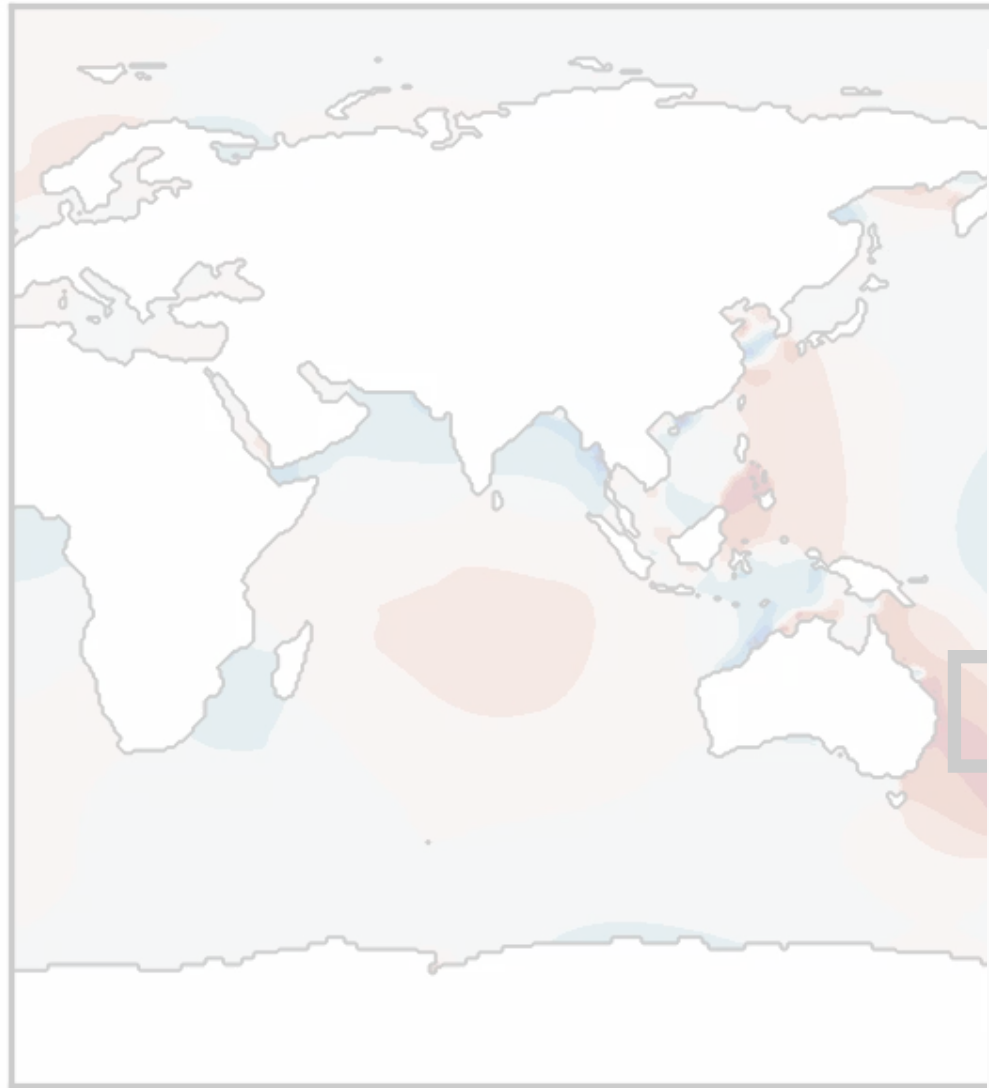
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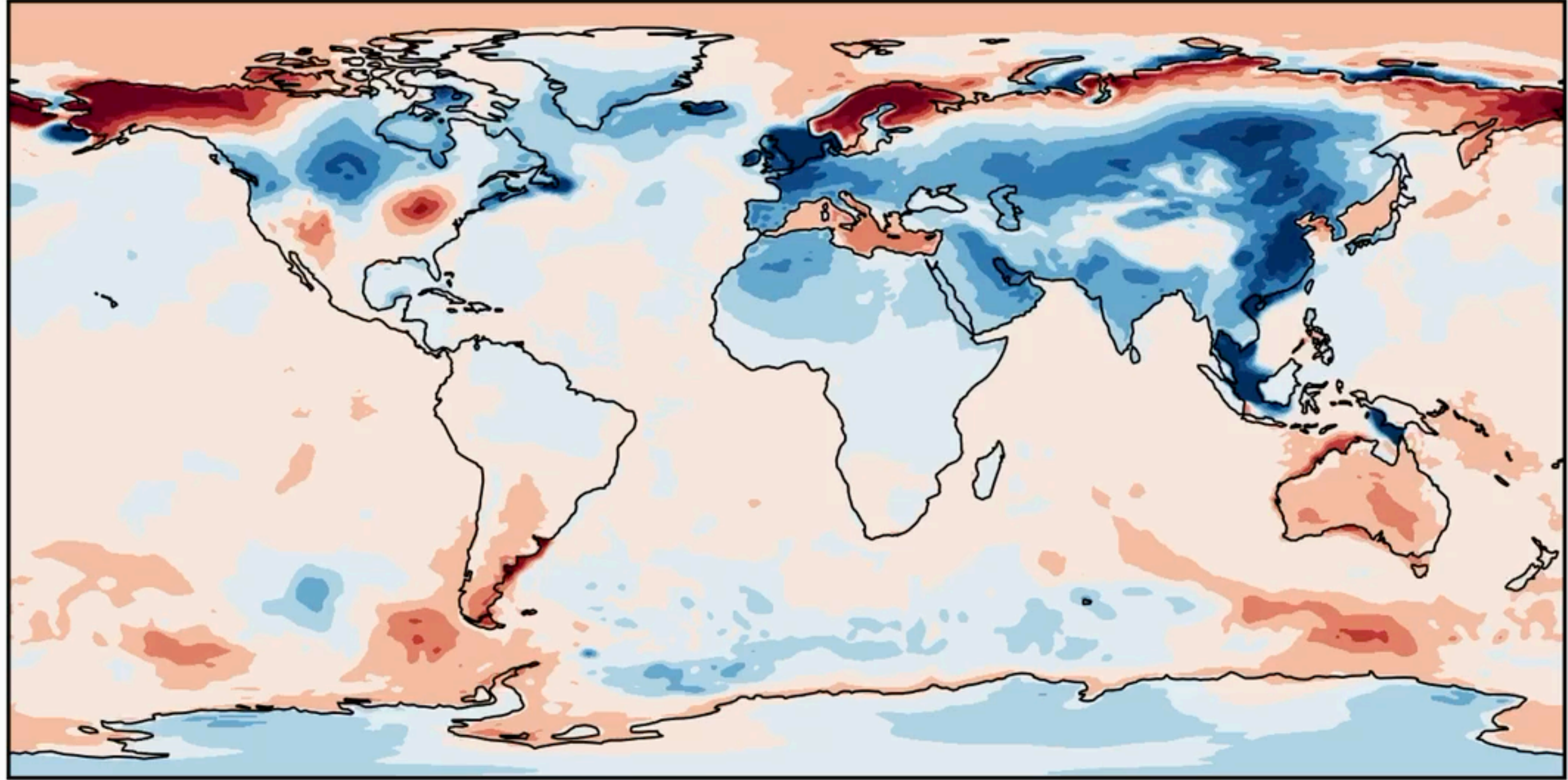
Example: M2 Tide & GRACE Groundtracks

M2 Tidal Signal



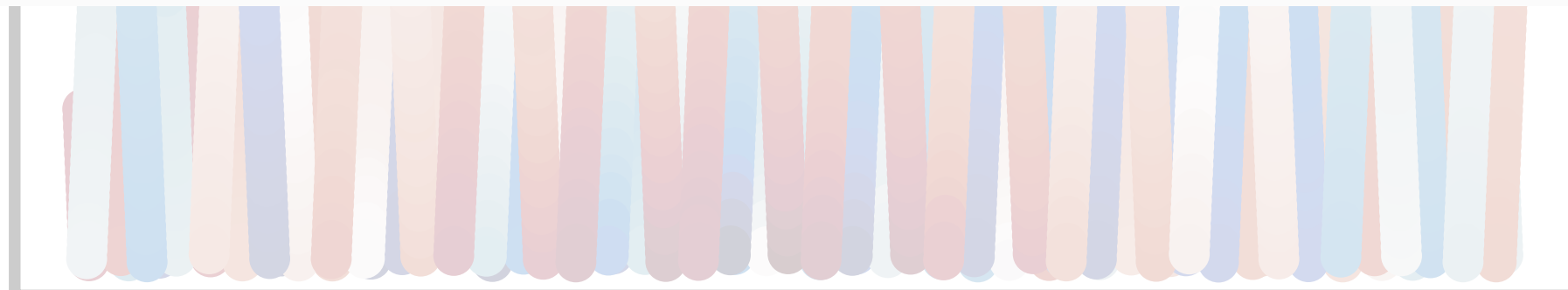
Non-Tidal AO Variability

2019-01-01 00:00:00 UTC

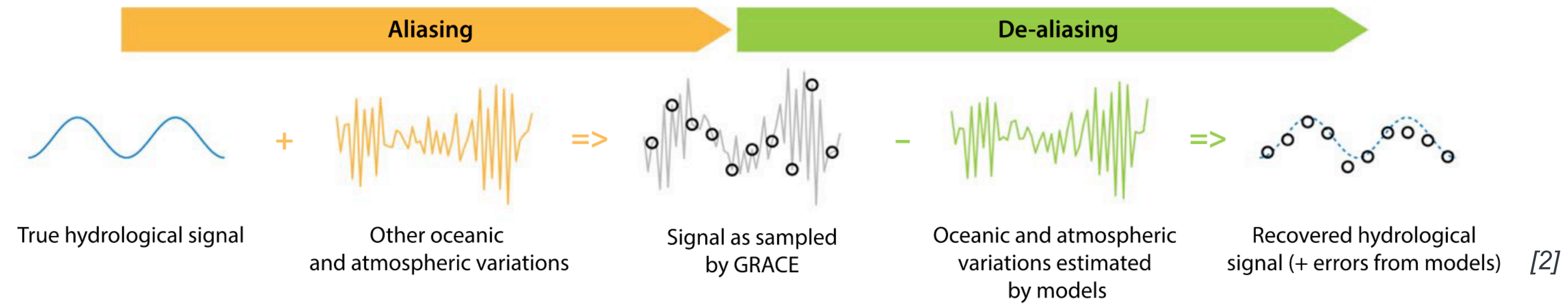


GRACE-FO Groundtrack

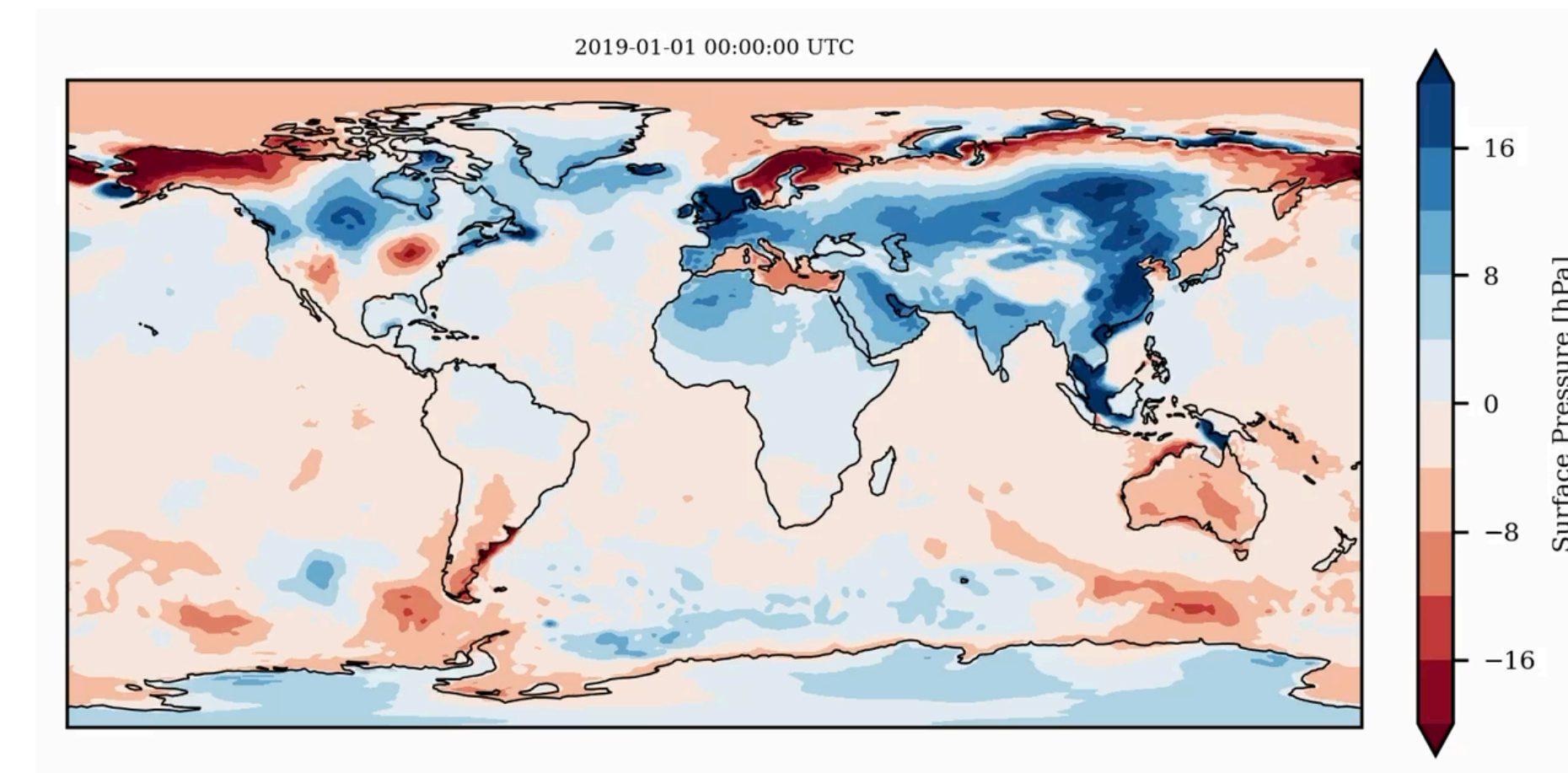
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Temporal Aliasing

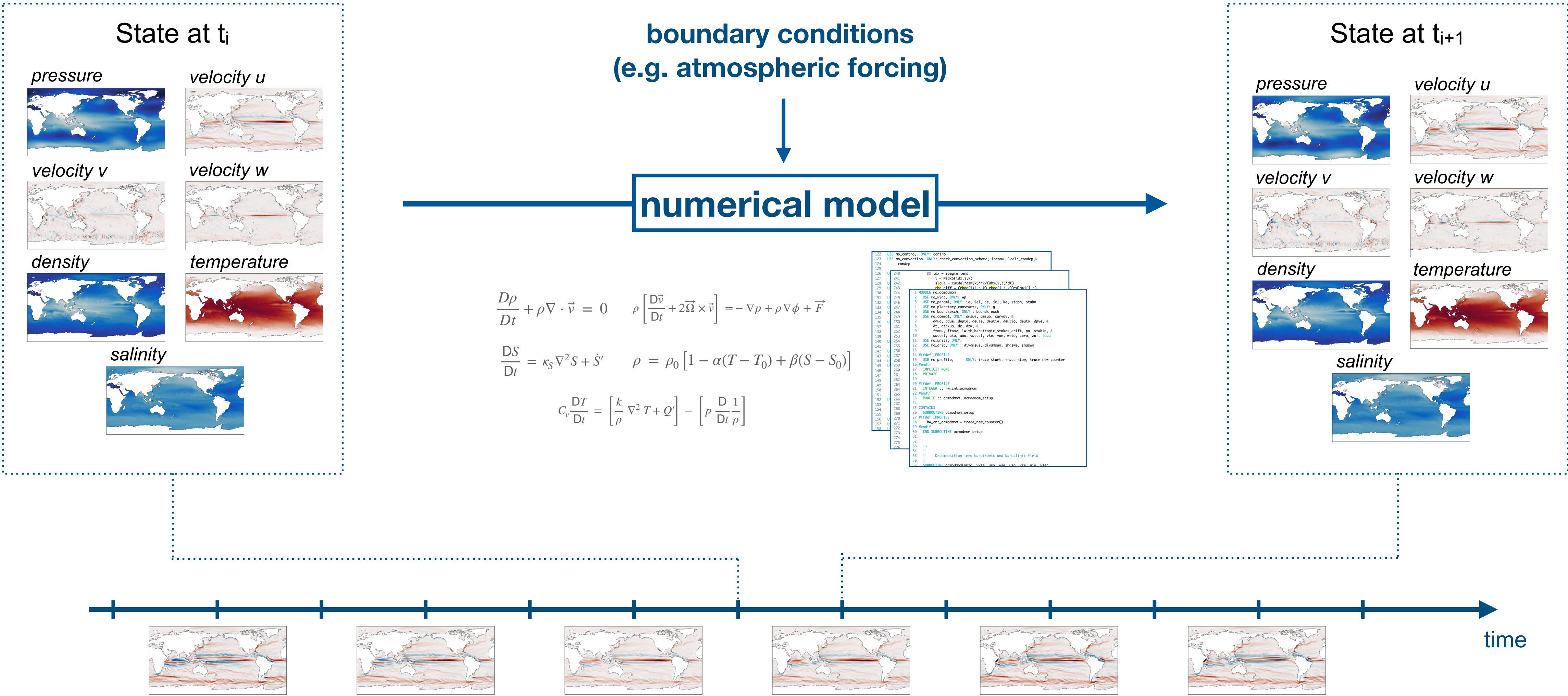


- High-frequency mass variations are modelled and supplied as background data products
- Subtracted in GRACE gravity field processing -> **de-aliasing**
- Non-tidal component provided by the Atmosphere and Ocean De-Aliasing Level-1B (**AOD1B**) data product
- Provides Stokes coefficients based on atmospheric and oceanic simulated mass variations

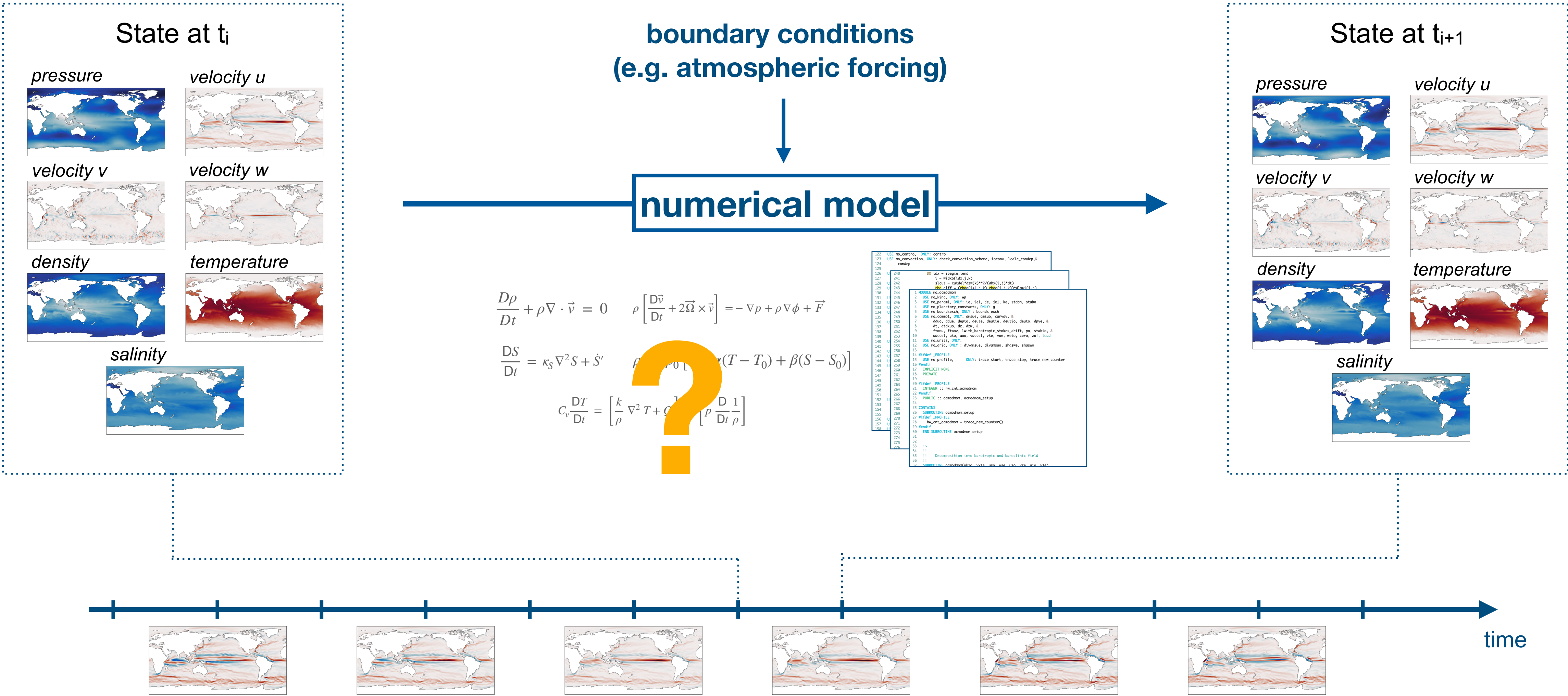


How do we model the atmosphere / ocean ?

How do we model the atmosphere / ocean ?



How do we model the atmosphere / ocean ?

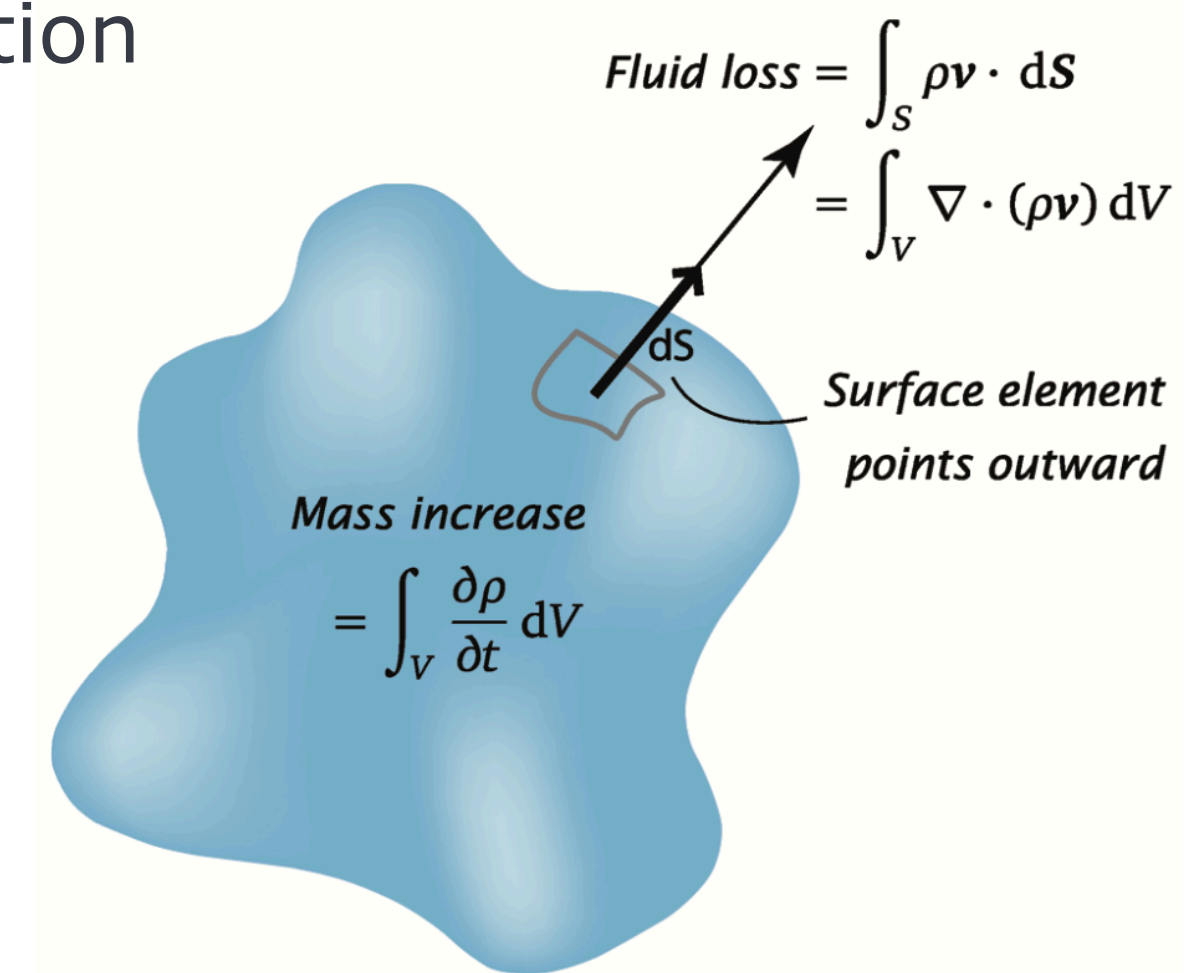


Fundamental Equations for Geophysical Flows

I. Continuity:

- In classical mechanics mass conservation is usually not *explicitly* considered
- For a fluid: density can change and mass conservation is one of the equations of motion

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0$$



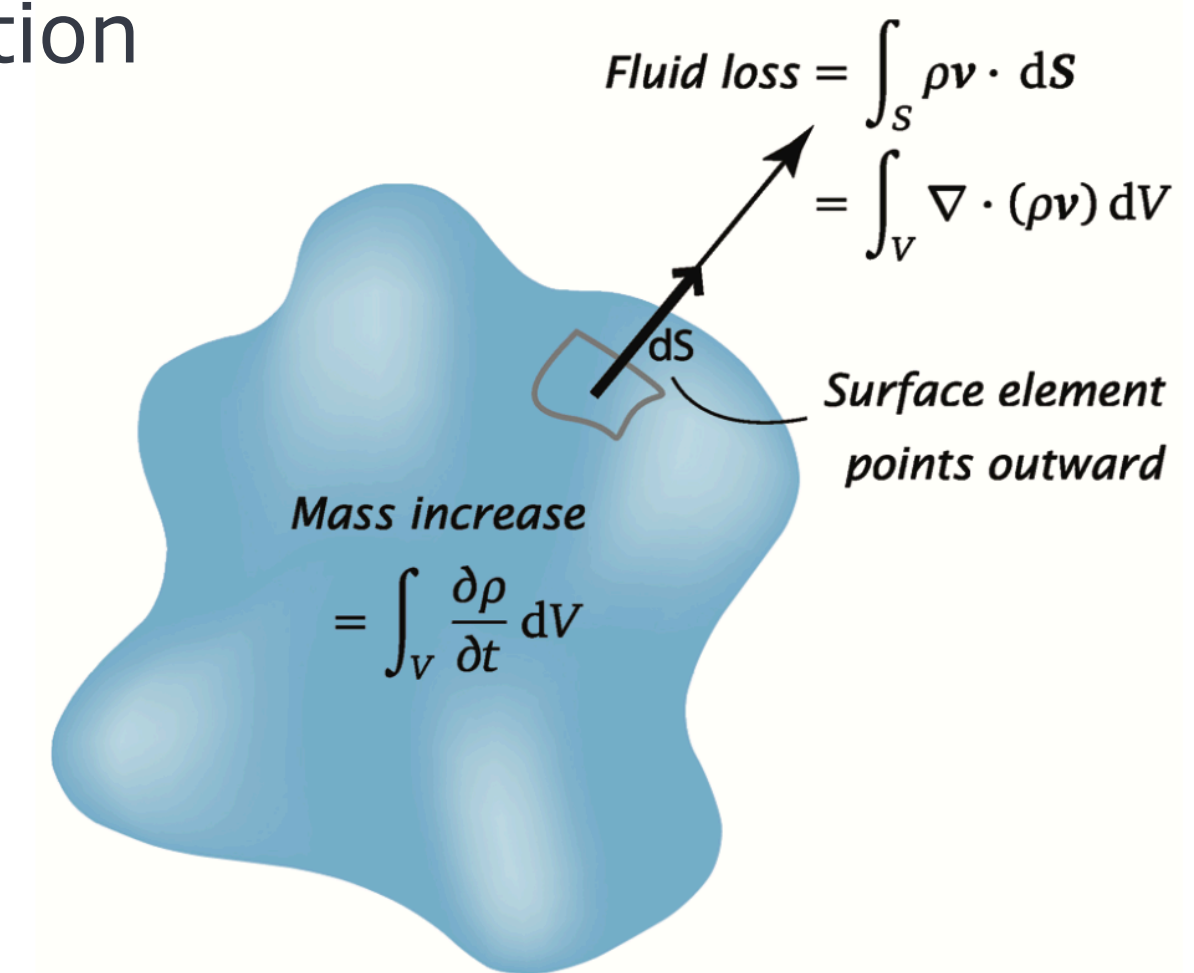
[3]

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$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0$$



Side-note: material derivative

- We often use the *material derivative* which describes changes of a property of a fluid parcel as it moves with the flow

- Combines local time-derivative and change due to movement (advection) $\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla) \rho$

- The continuity equation can thus be written: $\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0$

[3]

Fundamental Equations for Geophysical Flows

II. Momentum:

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \rho \nabla \phi + \vec{F}$$

pressure-gradient force

e.g. friction

*conservative forces,
e.g. gravity*

- How to account for friction on larger scales can be very difficult

Fundamental Equations for Geophysical Flows

II. Momentum:

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \rho \nabla \phi + \vec{F}$$

pressure-gradient force (pointing to $-\nabla p$)
e.g. friction (pointing to \vec{F})
conservative forces, e.g. gravity (pointing to $\rho \nabla \phi$)

- How to account for friction on larger scales can be very difficult
- We observe the atmosphere / ocean from the Earth and thus a rotating reference frame

$$\Rightarrow \rho \left[\frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} \right] = -\nabla p + \rho \nabla \phi + \vec{F}$$

- Rotational effects play a crucial role in GFD (see lecture on *Mass Change of the Oceans* tomorrow)

Fundamental Equations for Geophysical Flows

III. Equation of State:

- So far: 4 equations and 5 unknowns (\vec{v}, p, ρ)
- So far no distinction between atmosphere and ocean. Consider now the nature of the fluid
- How does density change with pressure, temperature, etc.

Fundamental Equations for Geophysical Flows

III. Equation of State:

- So far: 4 equations and 5 unknowns (\vec{v}, p, ρ)
- So far no distinction between atmosphere and ocean. Consider now the nature of the fluid
- How does density change with pressure, temperature, etc.

Atmosphere:

- For an ideal (dry) gas: $\rho = \frac{p}{RT}$
- While the composition of dry air is fairly uniform, water vapour can vary

$$\rho = \frac{p}{RT(1 + 0.608q)}$$

Ocean:

- Seawater varies mostly with temperature & salinity
- There is no equivalent to the ideal gas law
- Empirical (linear & simplified) equation of state:

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)]$$

Fundamental Equations for Geophysical Flows

IV. Energy Budget:

- The equation of state determines the relationship between pressure and density but it introduces new variables (temperature and salinity / humidity)
- Additional consideration: changes in the internal energy of the fluid \dot{e}

$$\frac{De}{Dt} = \overset{\text{rate of heat gain}}{Q} - \overset{\text{rate of work done}}{W}$$

$$C_v \frac{DT}{Dt} = \left[\frac{k}{\rho} \nabla^2 T + \overset{\text{internal heat sources}}{Q'} \right] - \left[p \frac{D}{Dt} \frac{1}{\rho} \right]$$

Heat gain through diffusion, i.e. neighboring parcels

- Internal heat sources (Q') not important for ocean but for atmosphere through evaporation and condensation

Fundamental Equations for Geophysical Flows

V. Salt & Moisture Budgets:

- **Ocean:** density varies with salinity
- **Atmosphere:** density varies with humidity

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S + \dot{S}'$$

diffusion of salinity

*sources / sinks
e.g. air-sea water
exchange, river
runoff, ice formation*

- On a high level, both budgets are similar
- Sources and sinks of salinity mainly confined to boundaries
- In the atmosphere, phase changes need to be considered which depend heavily on temperature
- Variability of water content in the atmosphere considerably higher

Fundamental Equations for Geophysical Flows

I. Continuity:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0$$

II. Momentum:

$$\Rightarrow \rho \left[\frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} \right] = -\nabla p + \rho \nabla \phi + \vec{F}$$

III. Equation of state:

$$\rho = \frac{p}{RT(1 + 0.608q)} \quad \rho = \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)]$$

IV. Energy budget:

$$C_v \frac{DT}{Dt} = \left[\frac{k}{\rho} \nabla^2 T + Q' \right] - \left[p \frac{D}{Dt} \frac{1}{\rho} \right]$$

V. Salt & moisture:

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S + \dot{S}'$$

Common Approximations

- The equations are rarely used without approximations to reduce complexity
- These can be quite different for the atmosphere and oceans

Typical Scales of Atmospheric and Oceanic Flows

Variable	Scale	Unit	Atmospheric value	Oceanic value
x, y	L	m	100 km = 10^5 m	10 km = 10^4 m
z	H	m	1 km = 10^3 m	100 m = 10^2 m
t	T	s	$\geq \frac{1}{2}$ day $\simeq 4 \times 10^4$ s	≥ 1 day $\simeq 9 \times 10^4$ s
u, v	U	m/s	10 m/s	0.1 m/s
w	W	m/s		
p	P	$\text{kg m}^{-1} \text{s}^{-2}$		variable
ρ	$\Delta\rho$	kg/m^3		

[4]

1. Hydrostatic Approximation:

- Horizontal scales much larger than vertical scales
- Vertical pressure gradient balanced by weight of fluid

$$\rho \left[\frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} \right] = -\nabla p + \rho \nabla \phi + \vec{F}$$

$\xrightarrow{\text{z-component}}$

$$\frac{\partial p}{\partial z} = -\rho g$$

2. Shallow-Fluid Approximation:

- Vertical dimension of ocean & atmosphere small compared to Earth radius

Common Approximations

3. Boussinesq Approximation:

- Mainly used in ocean general circulation models
- Assumes variations in density are small compared to mean density

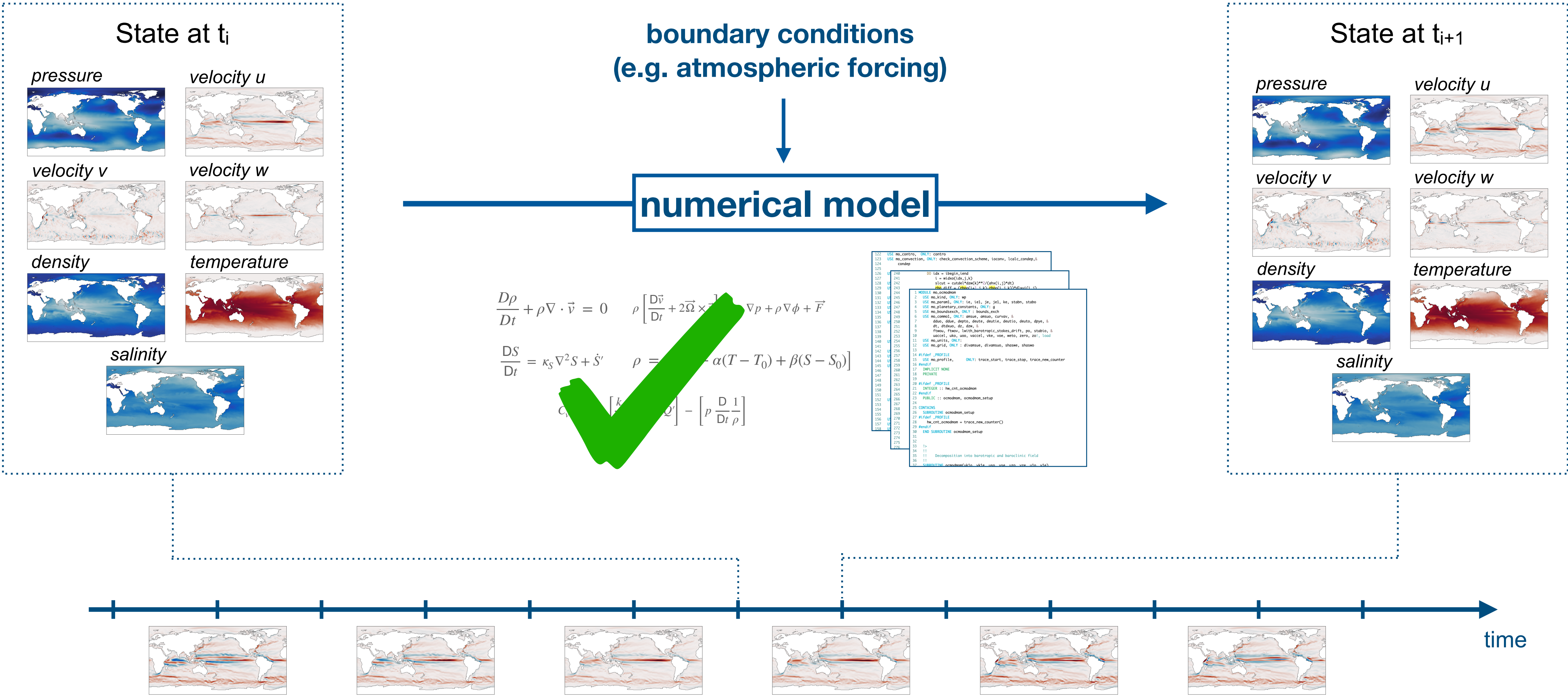
$$\rho = \rho_0 + \rho' \quad |\rho'| \ll \rho_0$$

- Simplifies equations in several ways, but most importantly the continuity eq.:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0 \quad \rightarrow \quad \nabla \cdot \vec{v} = 0$$

- Conservation of mass becomes conservation of volume!
- In applications where ocean mass is relevant (i.e. GRACE contexts), Boussinesq ocean model data often needs to be corrected!

How do we model the atmosphere / ocean ?



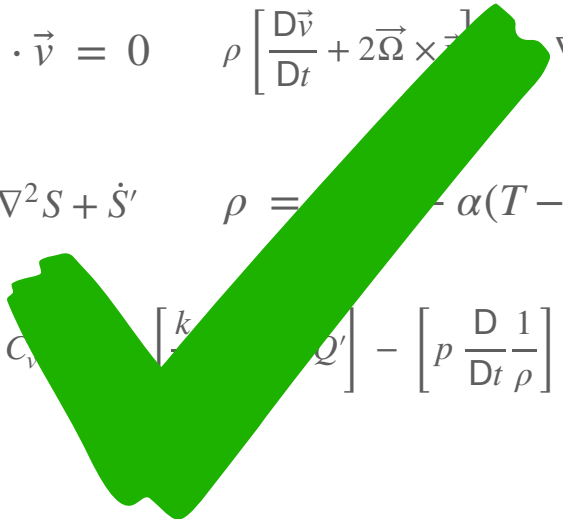
$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0$$

$$\rho \left[\frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} \right] = -\nabla p + \rho \nabla \phi + \vec{F}$$

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S + \dot{S}'$$

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)]$$

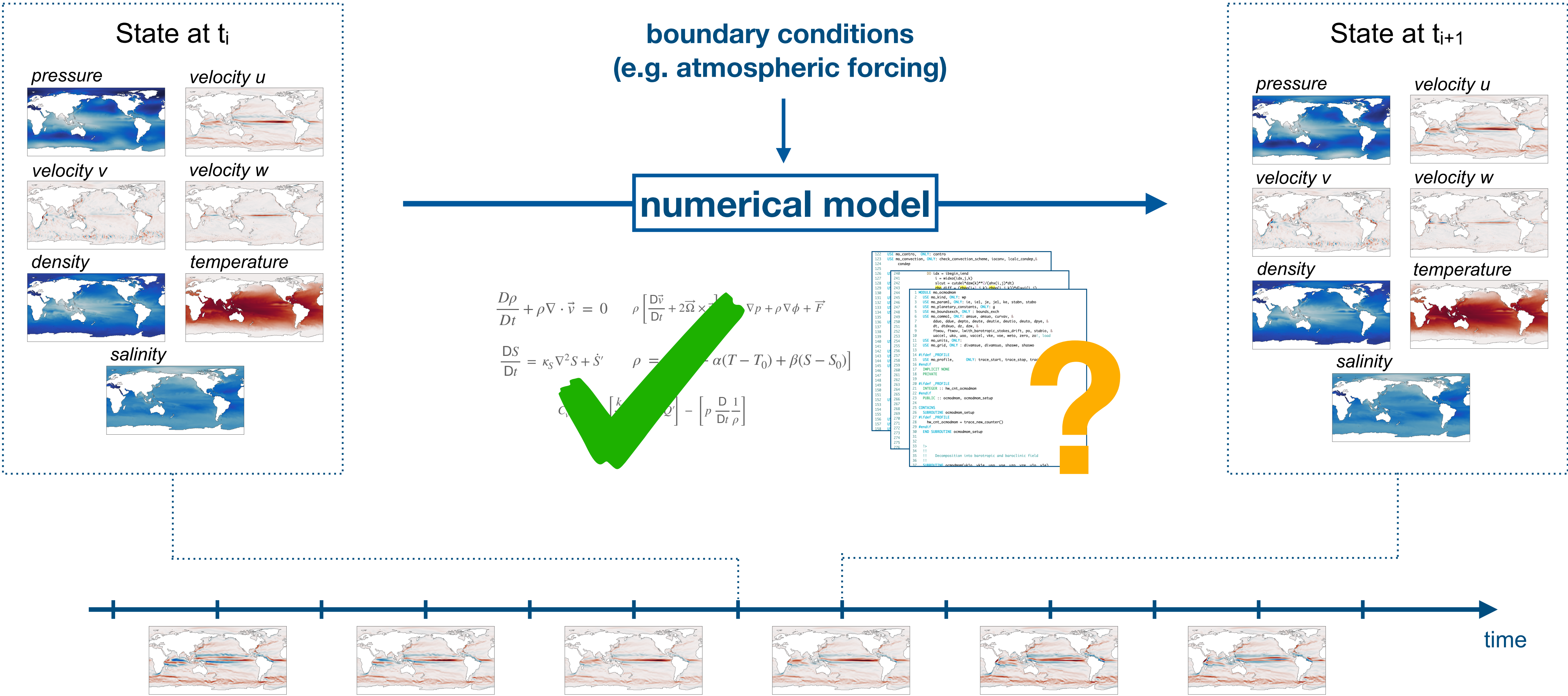
$$c_p \left[k \frac{D\psi}{Dt} \right] - \left[p \frac{D}{Dt} \frac{1}{\rho} \right]$$



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123 USE mo_convection, ONLY: check_convection_scheme, locam, localc_condep, locdep
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How do we model the atmosphere / ocean ?



$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0 \quad \rho \left[\frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} \right] = -\nabla p + \rho \nabla \phi + \vec{F}$$

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S + \dot{S}' \quad \rho = \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)]$$

$$c_p \left[k \frac{D\psi}{Dt} \right] - \left[p \frac{D}{Dt} \frac{1}{\rho} \right]$$

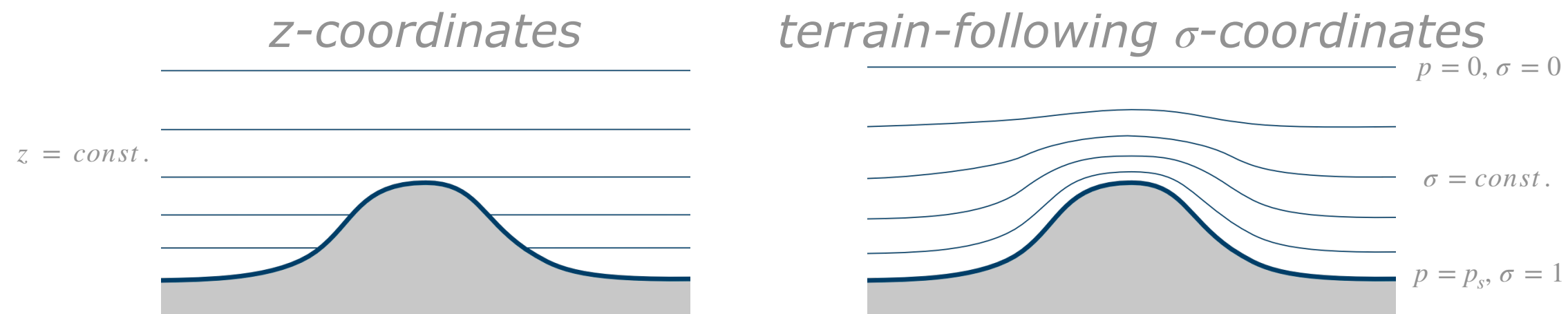
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123 USE mo_convection, ONLY: check_convection_scheme, locam, localc_condep, locdep
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A Few Points on Numerical Implementations:

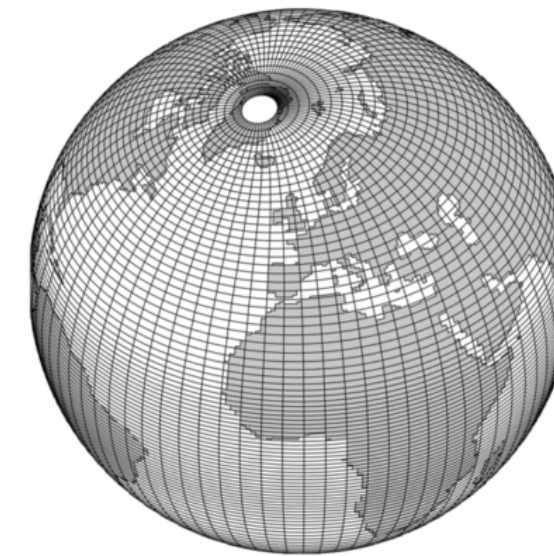
- How the geophysical equations are implemented plays a huge role in the applicability of the model
- These are just a few examples of many:

Vertical Coordinates:

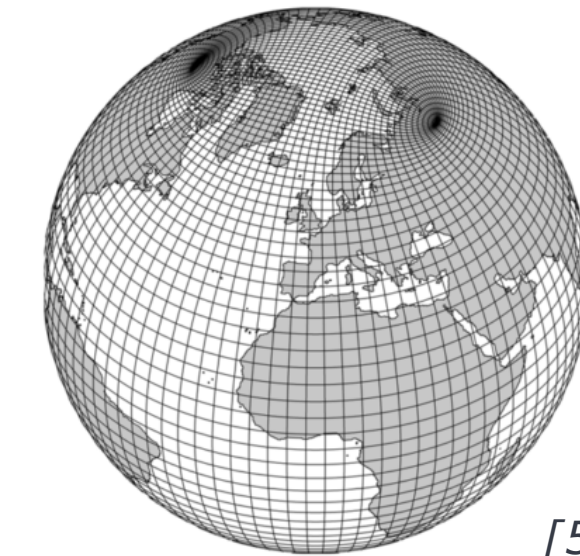


Horizontal Grids:

regular grid

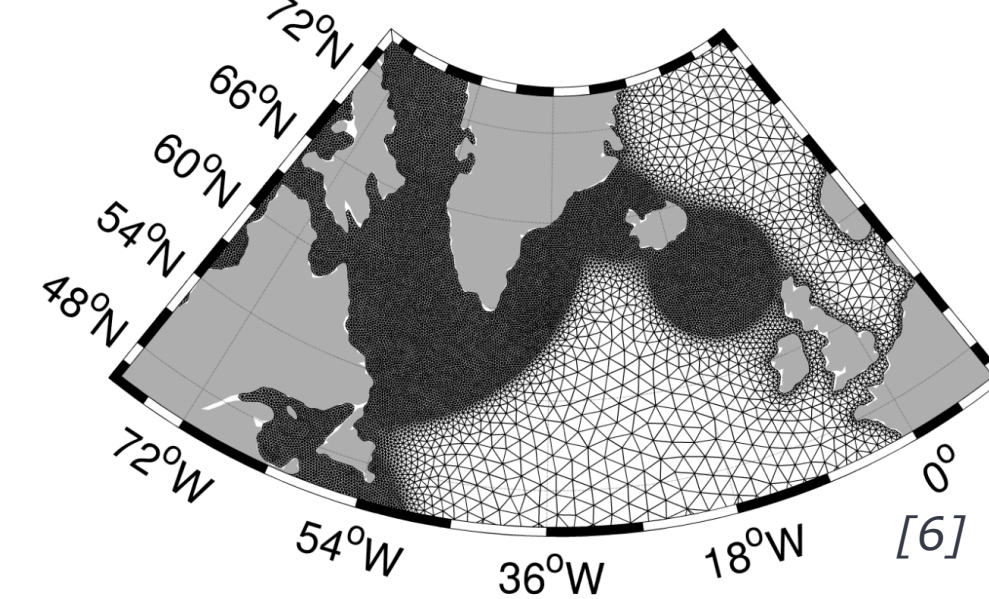


curvilinear grid



[5]

unstructured meshes

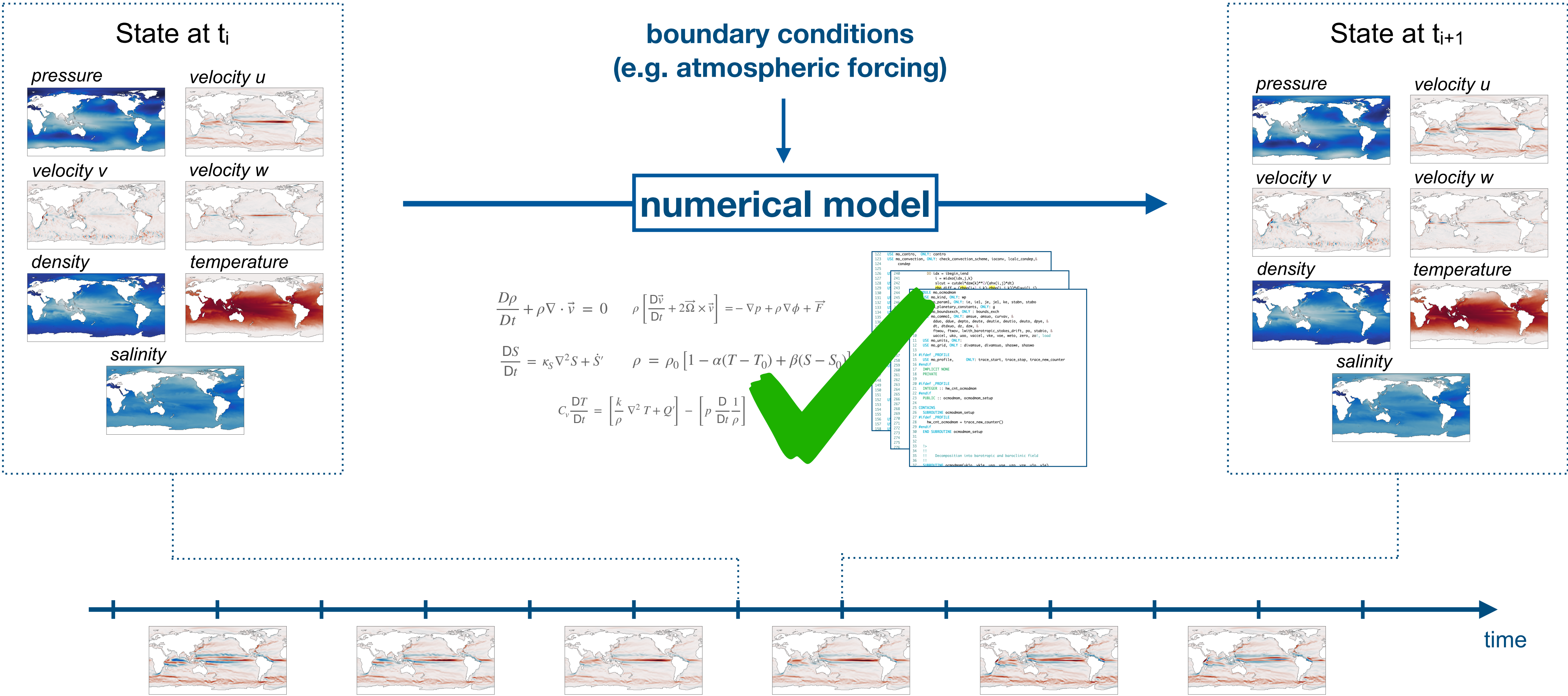


[6]

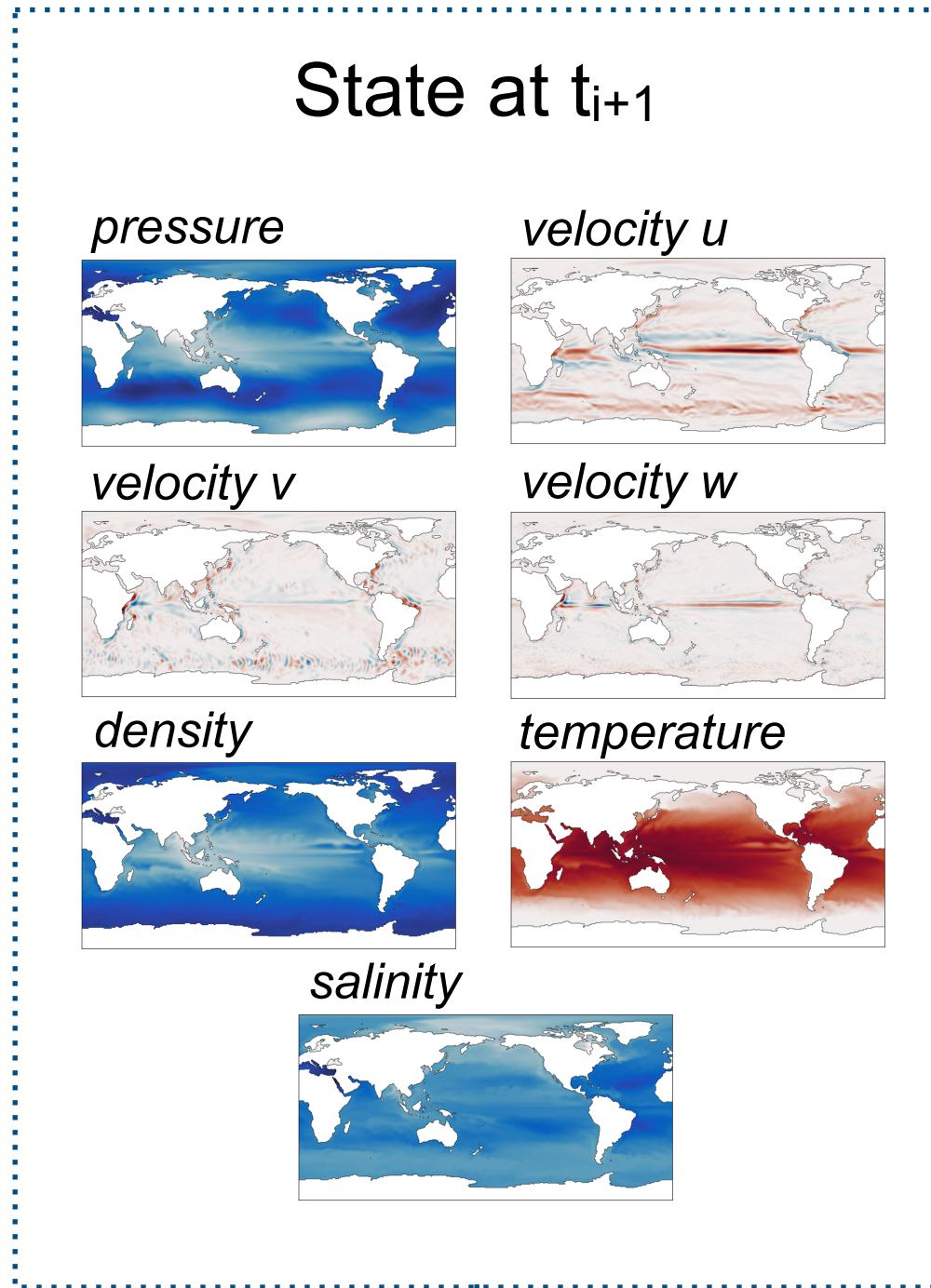
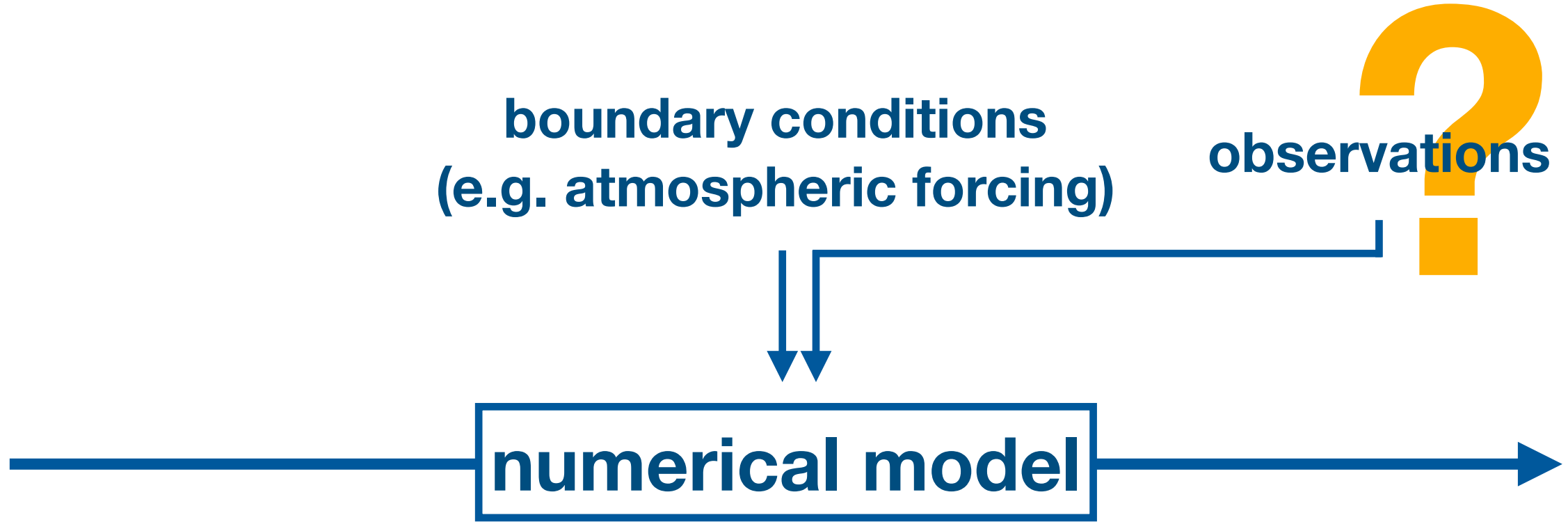
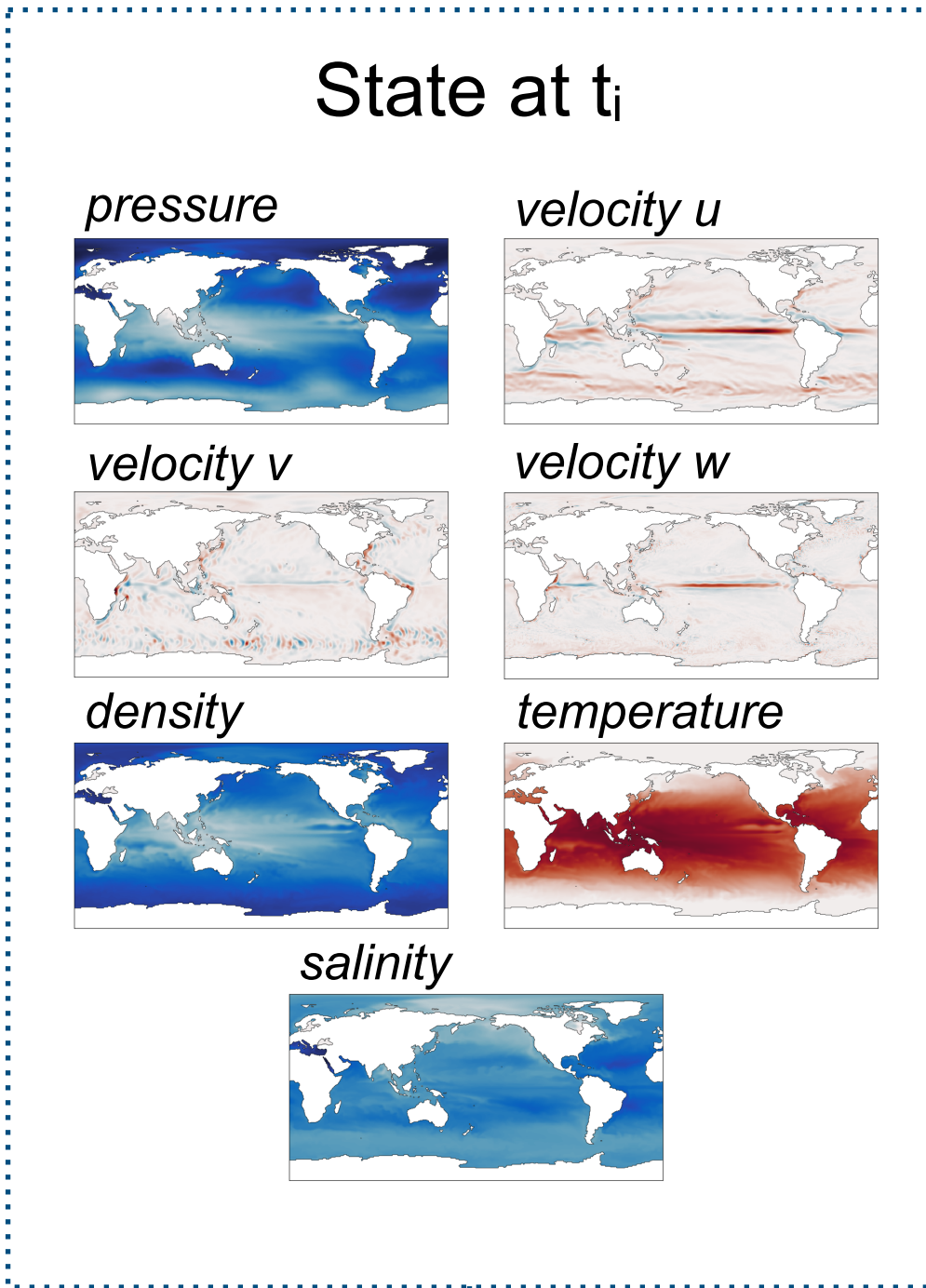
Subgrid processes:

- Many processes occur on scales that are not resolvable by the model but are important for accurate representation of the dynamics
- Impact included through parametrization that describe their net effect on the flow
- Examples: turbulent friction, boundary interactions, cloud microphysics, ...

How do we model the atmosphere / ocean ?



How do we model the atmosphere / ocean ?



$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0 \quad \rho \left[\frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} \right] = -\nabla p + \rho \nabla \phi + \vec{F}$$

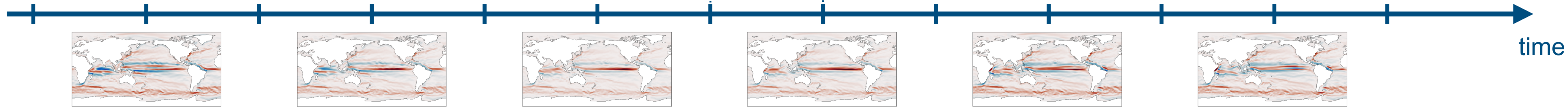
$$\frac{DS}{Dt} = \kappa_S \nabla^2 S + \dot{S}' \quad \rho = \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)]$$

$$C_v \frac{DT}{Dt} = \left[\frac{k}{\rho} \nabla^2 T + Q' \right] - \left[p \frac{D}{Dt} \frac{1}{\rho} \right]$$



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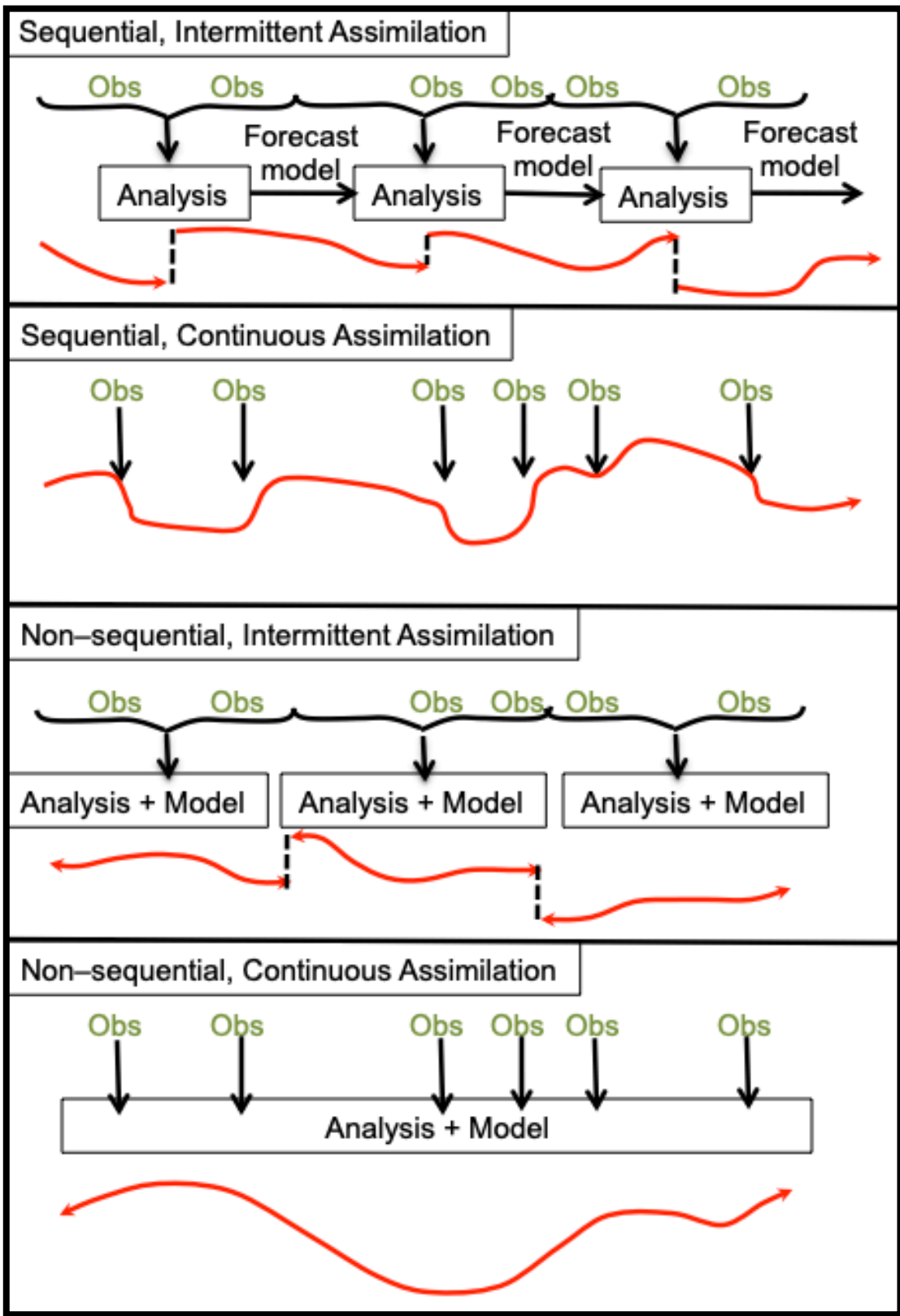
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Including observational data / data assimilation

- Measurement data can be introduced into models through data assimilation to create the best estimate of the atmospheric / oceanic state

Approaches to data assimilation:



[7]

Sequential assimilation:

only considers observation made in the past until the time of analysis

Non-sequential:

considers observations made before and after the nominal time of the analysis (e.g. 4D-var)

Retrospective assimilation:

observation from the future can be used (e.g reanalysis)

Intermittent:

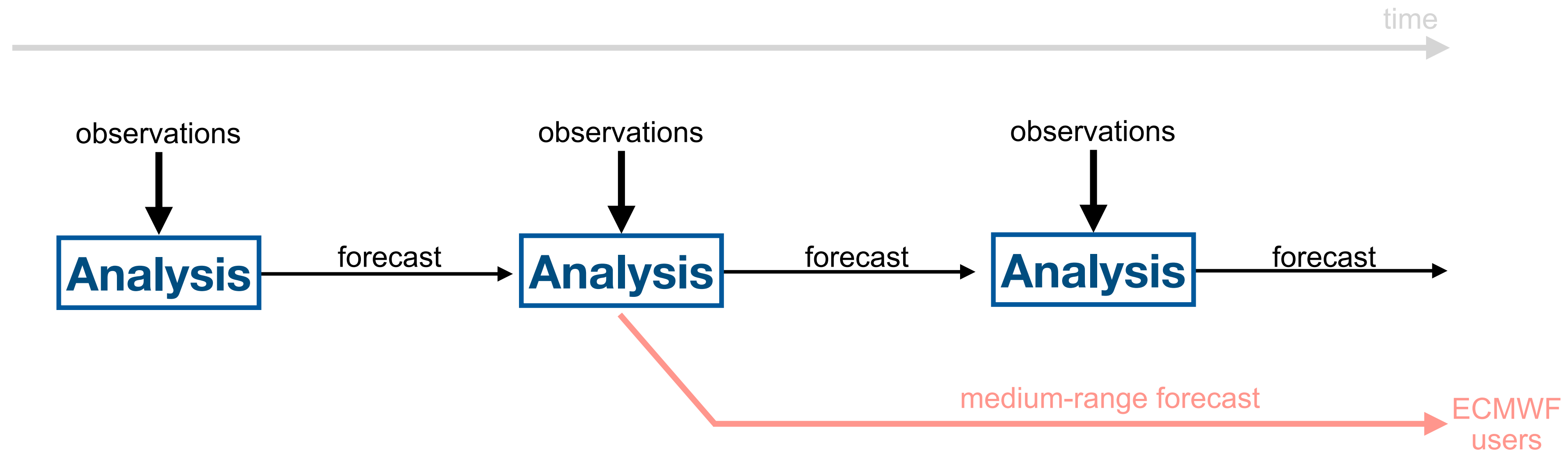
observations can be processed in small batches - often leads to abrupt changes

Continuous:

observation batches over longer periods are considered

Example: ECMWFs 4D-Var

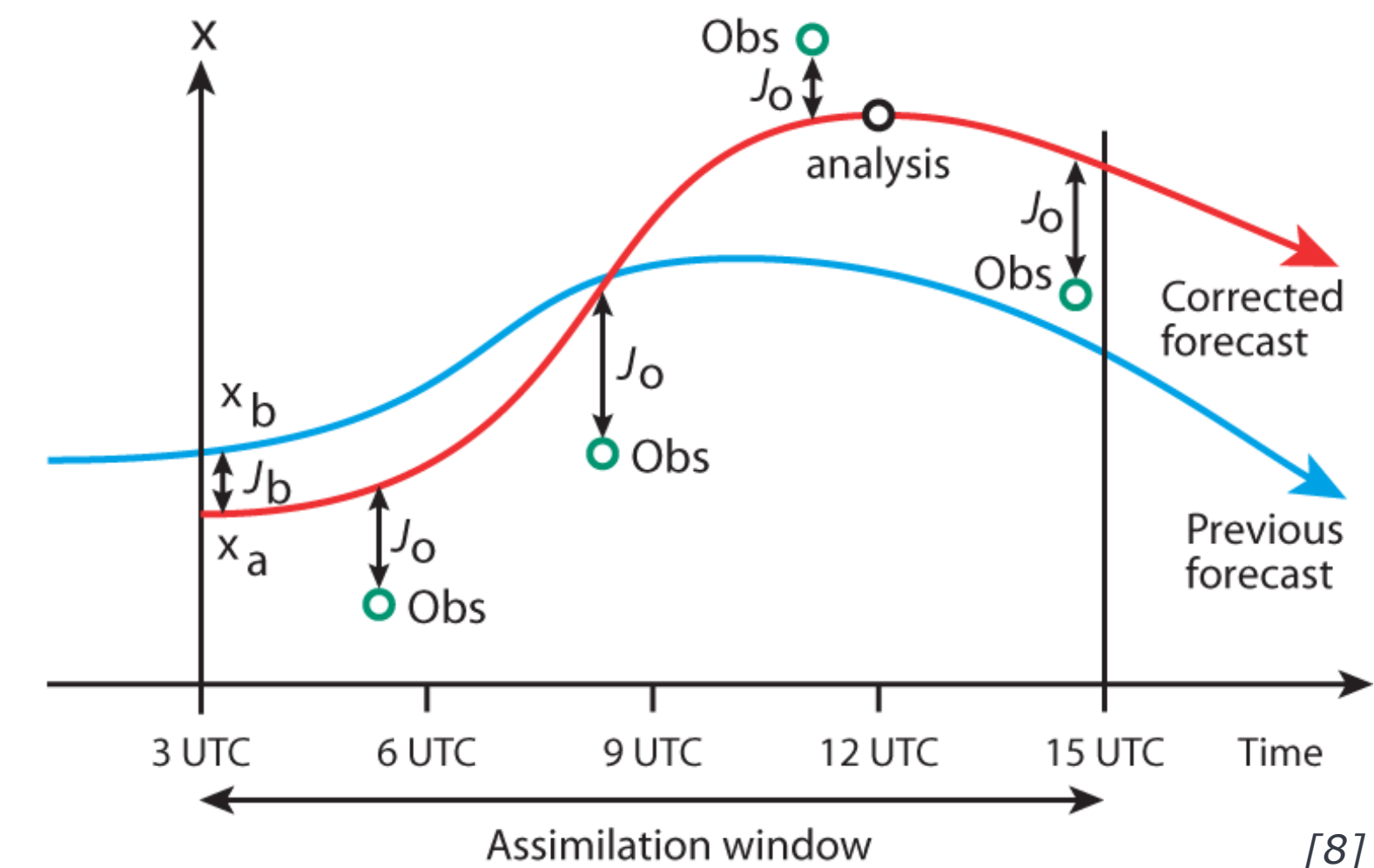
- Goal: produce the 'optimal' initial conditions (analysis) for numerical forecasts



- Based on an analysis (i.e. initial conditions) a short forecast is made
- Observations are used to correct the forecast and produce a new analysis
- From an analysis, longer forecasts are produced and disseminated to users

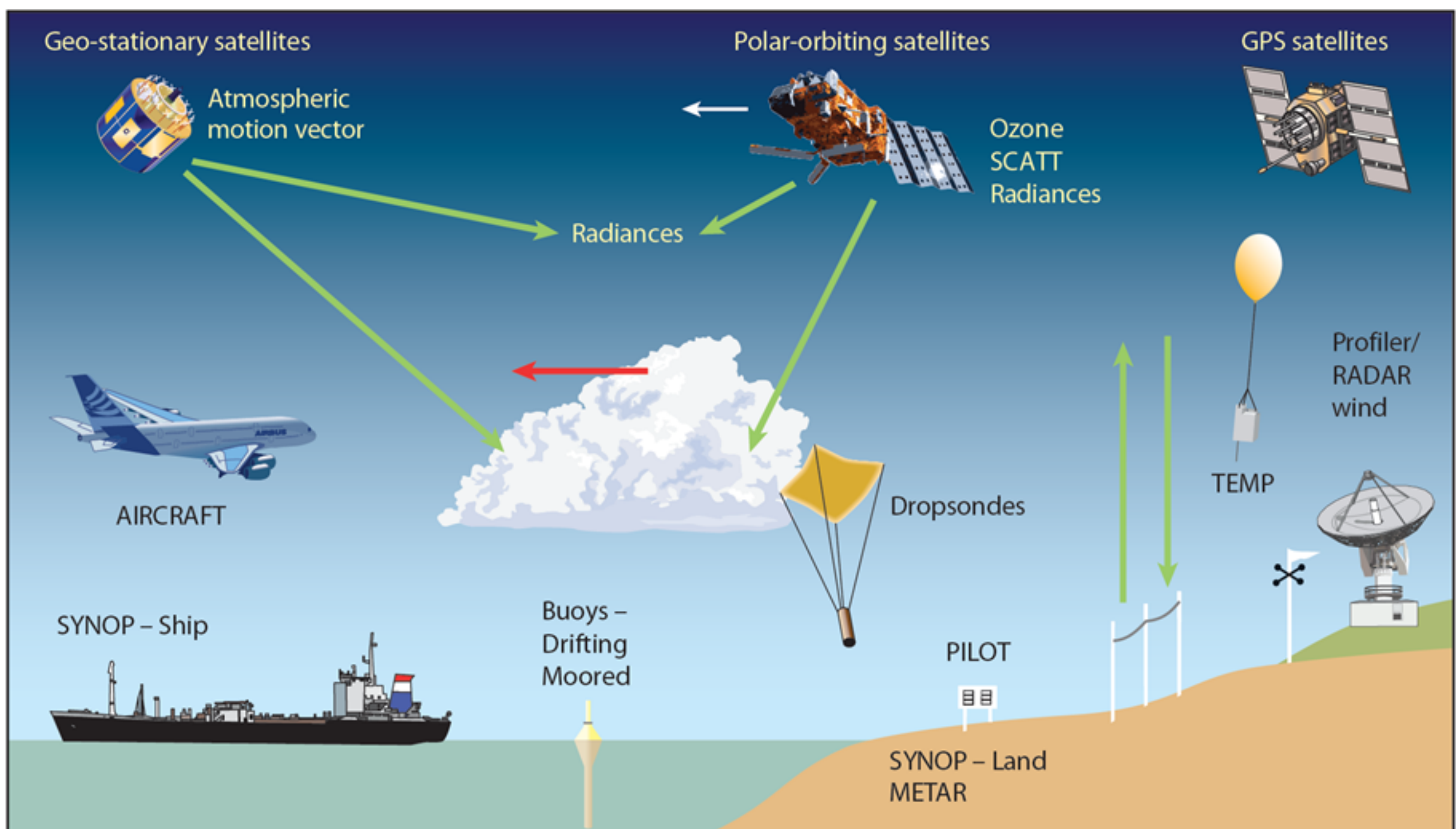
Example: ECMWFs 4D-Var

- Short-range forecast (12 hours) produced from previous analysis
- Calculate forecast-based 'observation predictions'
 - For in-situ observation: interpolate the model variable to observation location
 - For remote sensing: calculate observed quantities from model variables
- Adjust forecast to find best compromise between initial prediction (J_b) and observations (J_o)

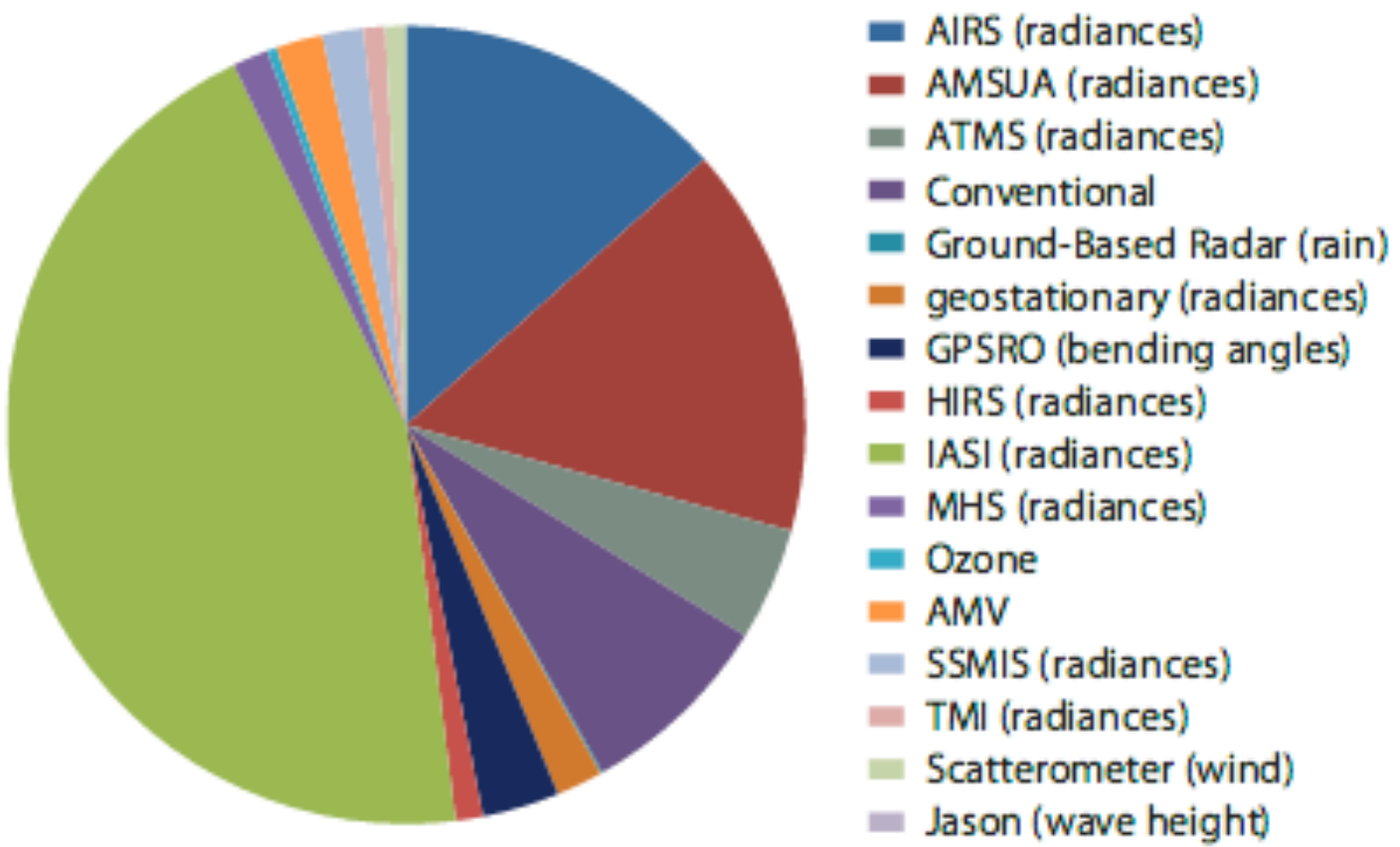


Example: ECMWFs 4D-Var

Observation sources



[9]

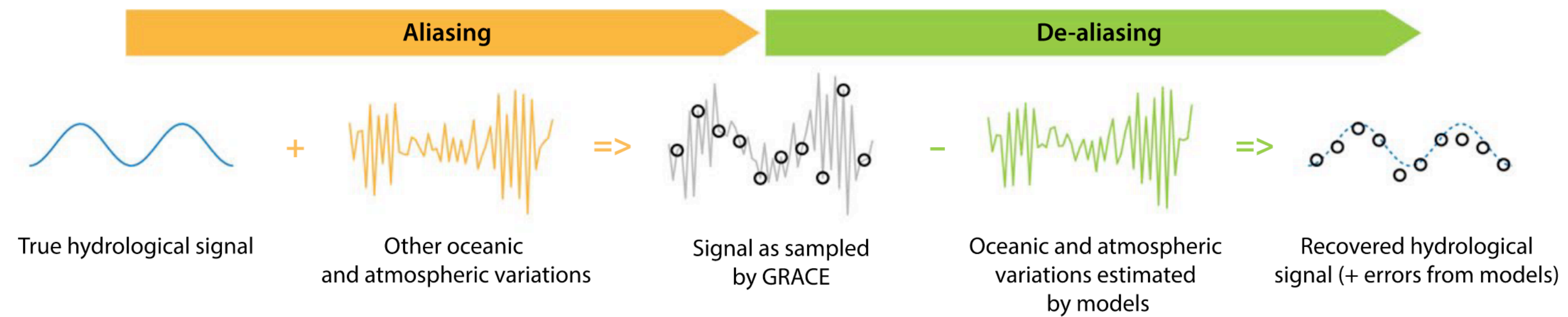


[10]

- Majority of observations are satellite based (largely radiances)
- ~ 60 million observations are available each day for assimilation

[9] https://www.ecmwf.int/sites/default/files/obs_inputs.png
 [10] R. Owens, T. Hewson, ECMWF Forecast User Guide, Section 2.4, 10.21957/m1cs7h

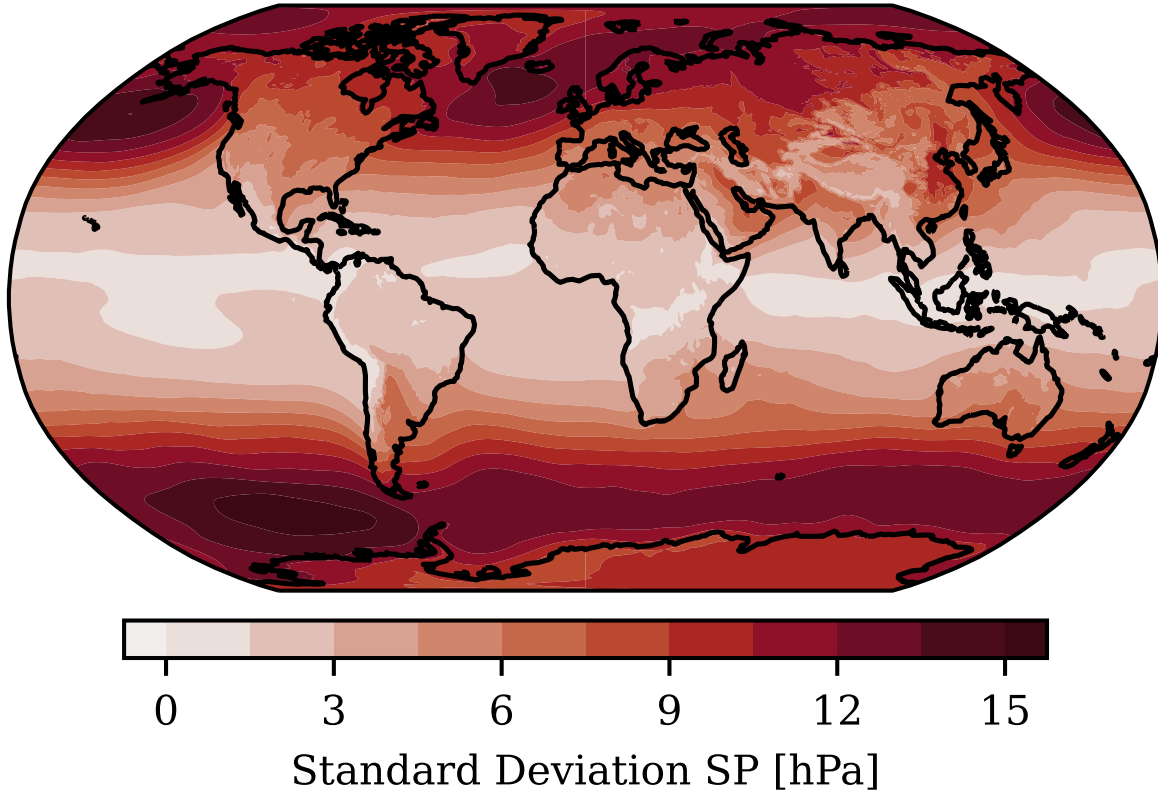
Back to GRACE Background Modeling



[2]

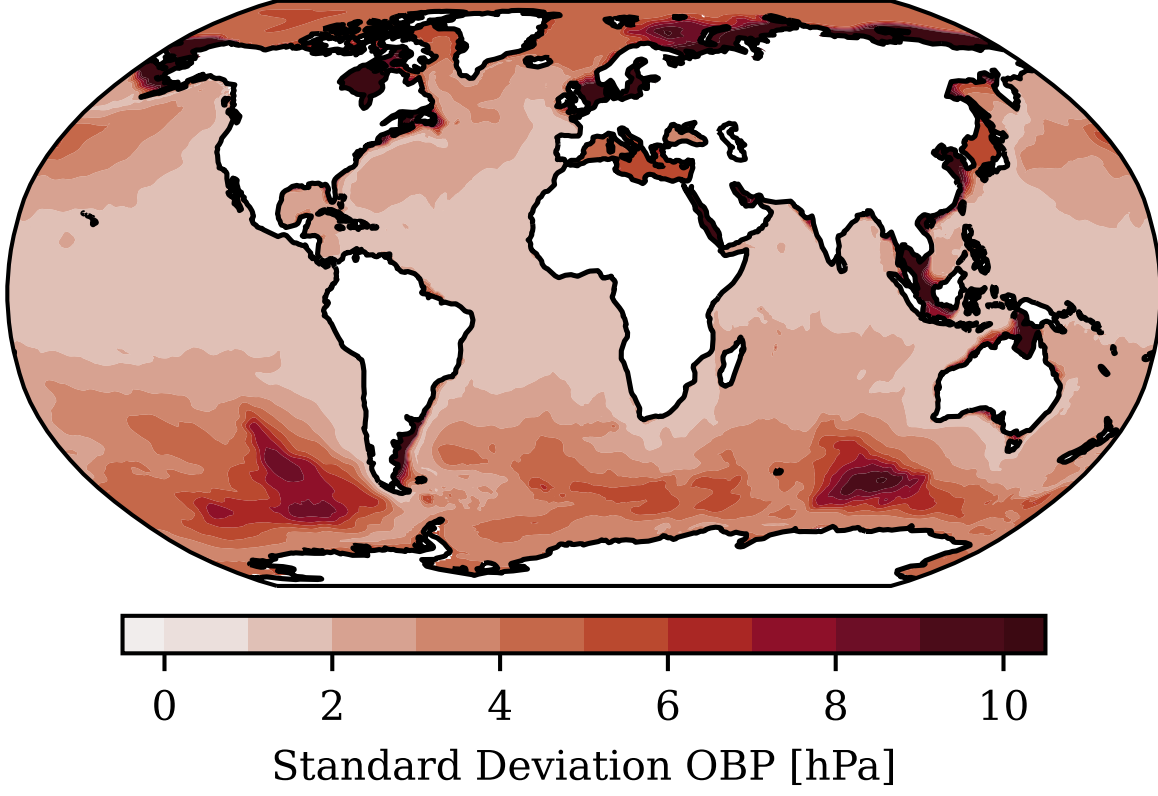
AOD1B Components

Atmospheric Surface Pressure:



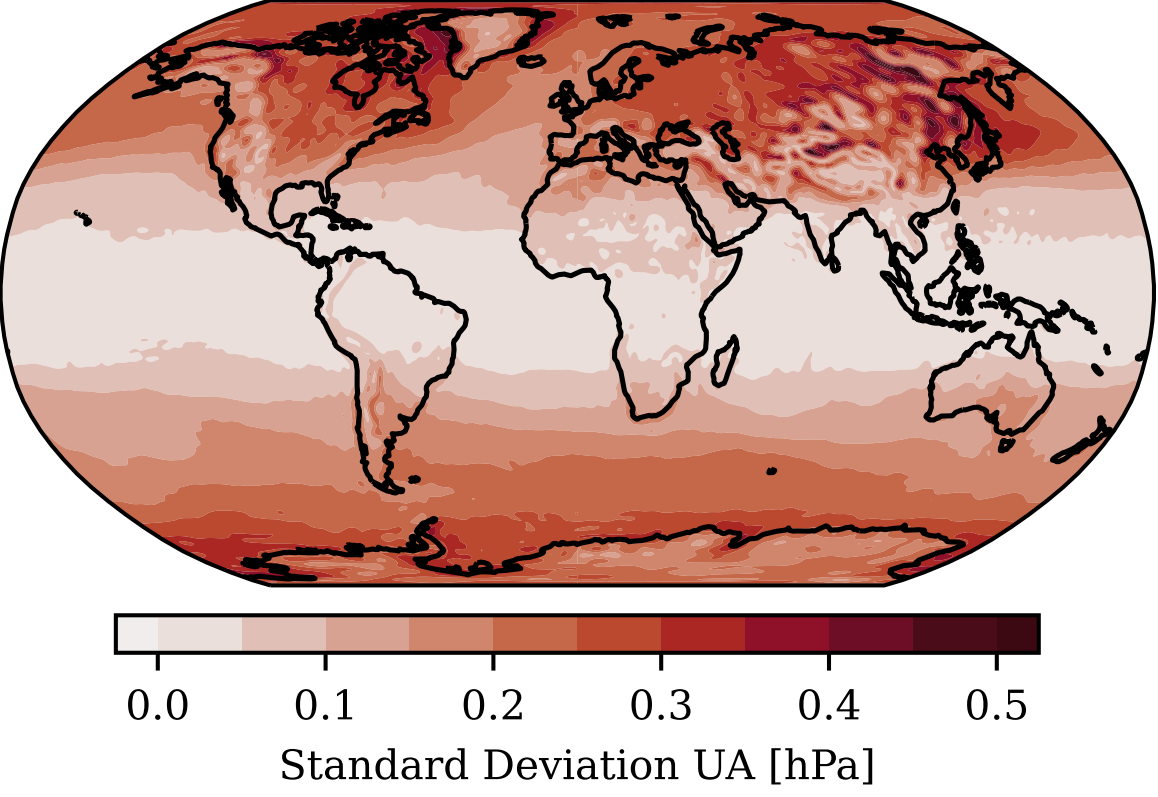
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Ocean Bottom Pressure:



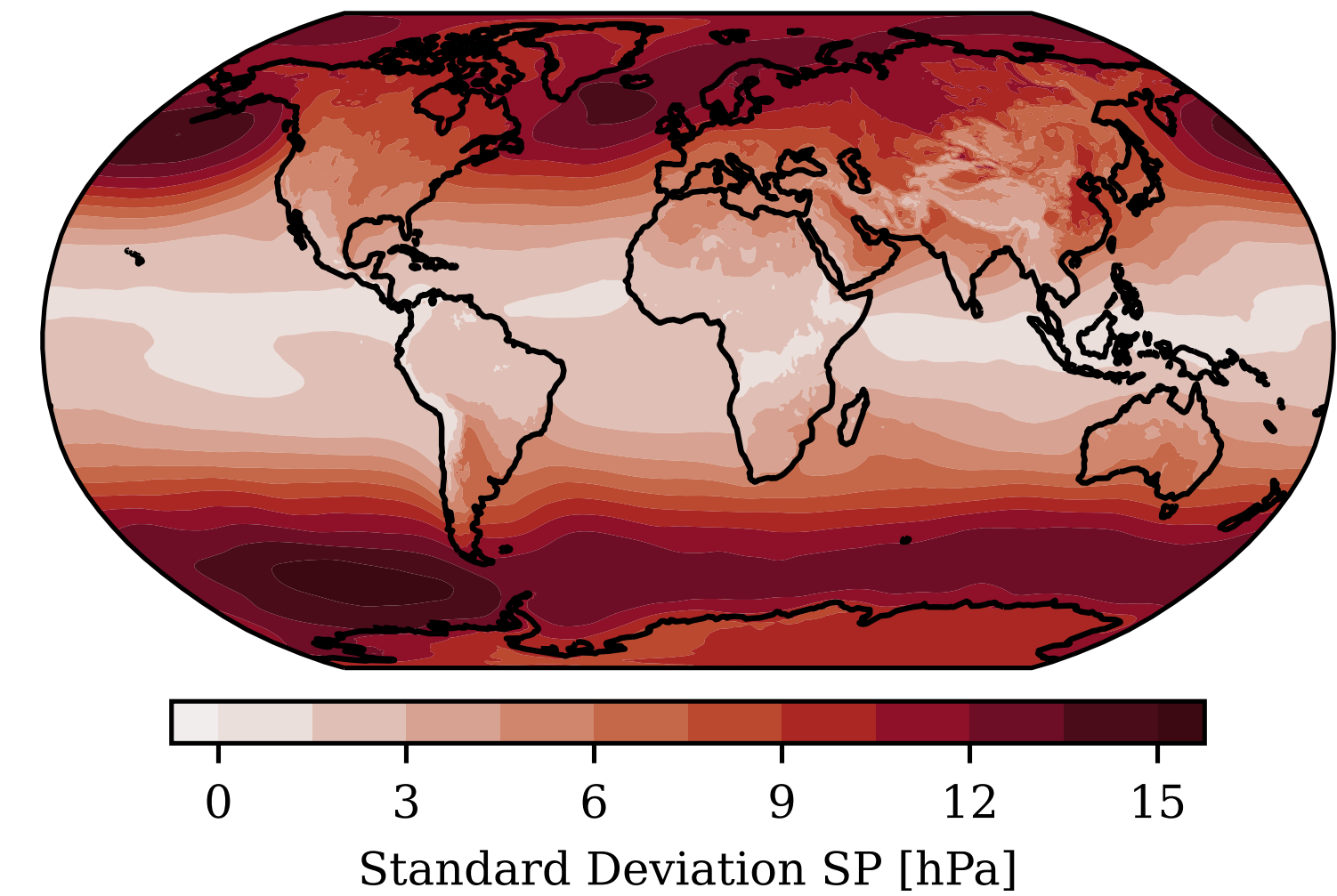
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Upper Air Variations:

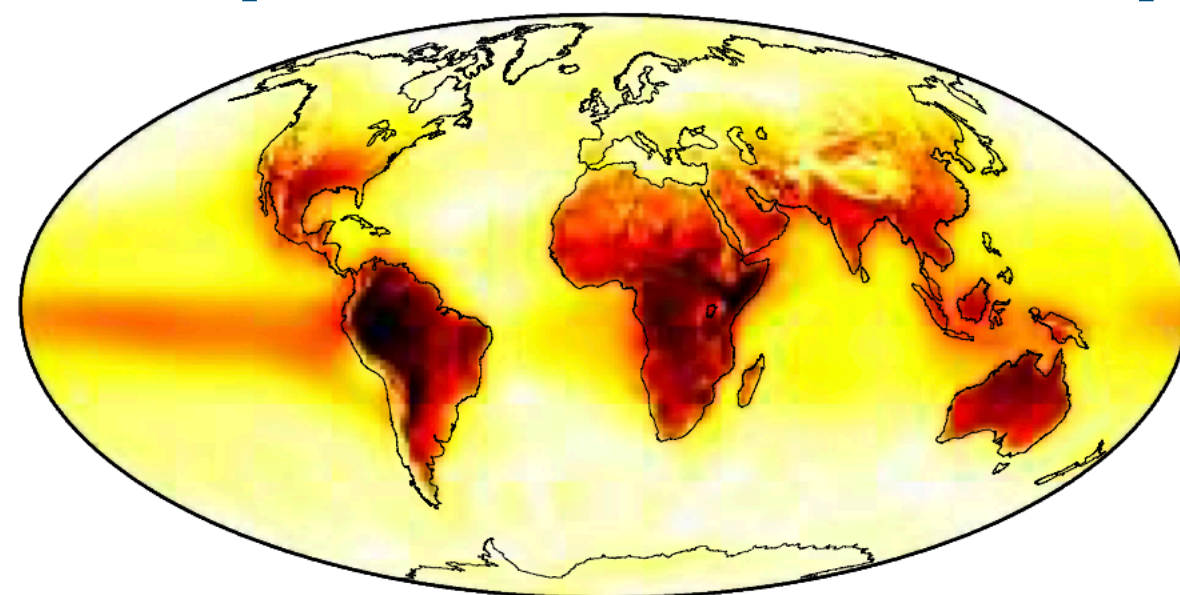


AOD1B Components: Surface Pressure

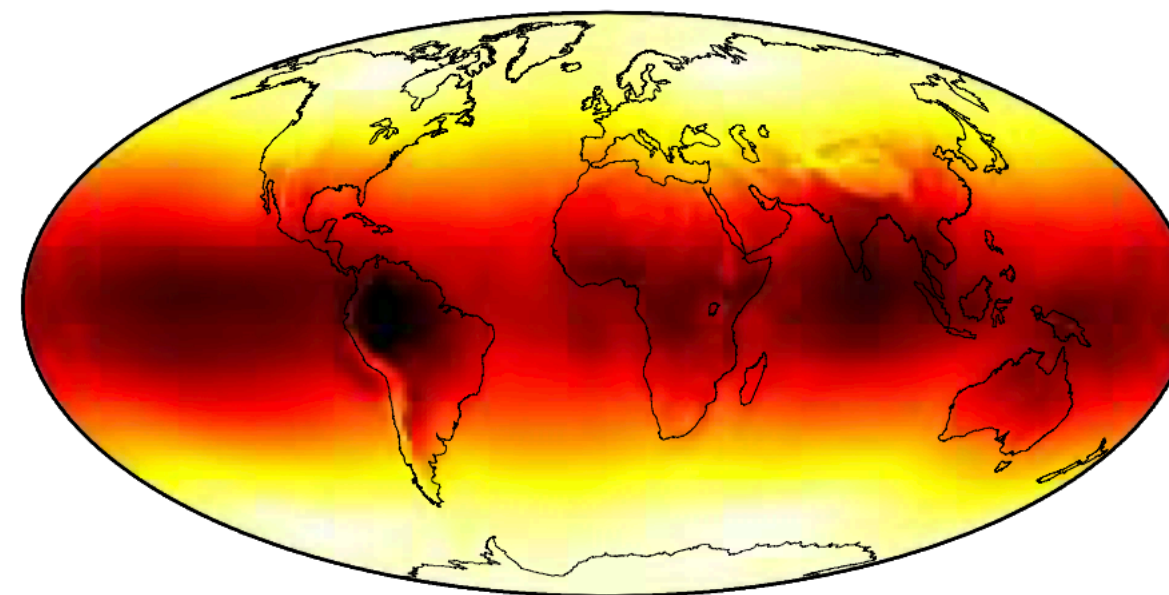
- Based on ECMWFs ERA5 reanalysis and ECMWF IFS data for operational processing
- Contains tidal signals driven by:
 - Solar radiation (S_1 & S_2)
 - Ocean tides (i.e. variations of the lower boundary, e.g. M_2)
- Tidal signals are estimated and removed
- Generally very good representation due to high weight of barometer data in ECMWFs DA



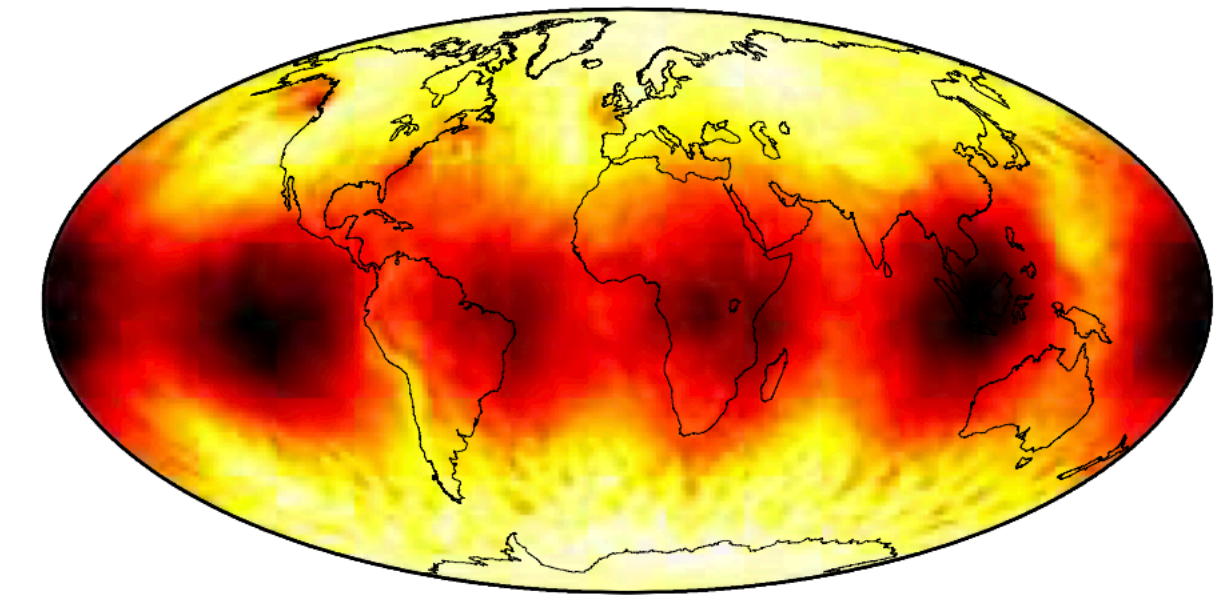
Example Amplitudes of Atmospheric Tides:



0 40 80 120 S_1 [Pa]



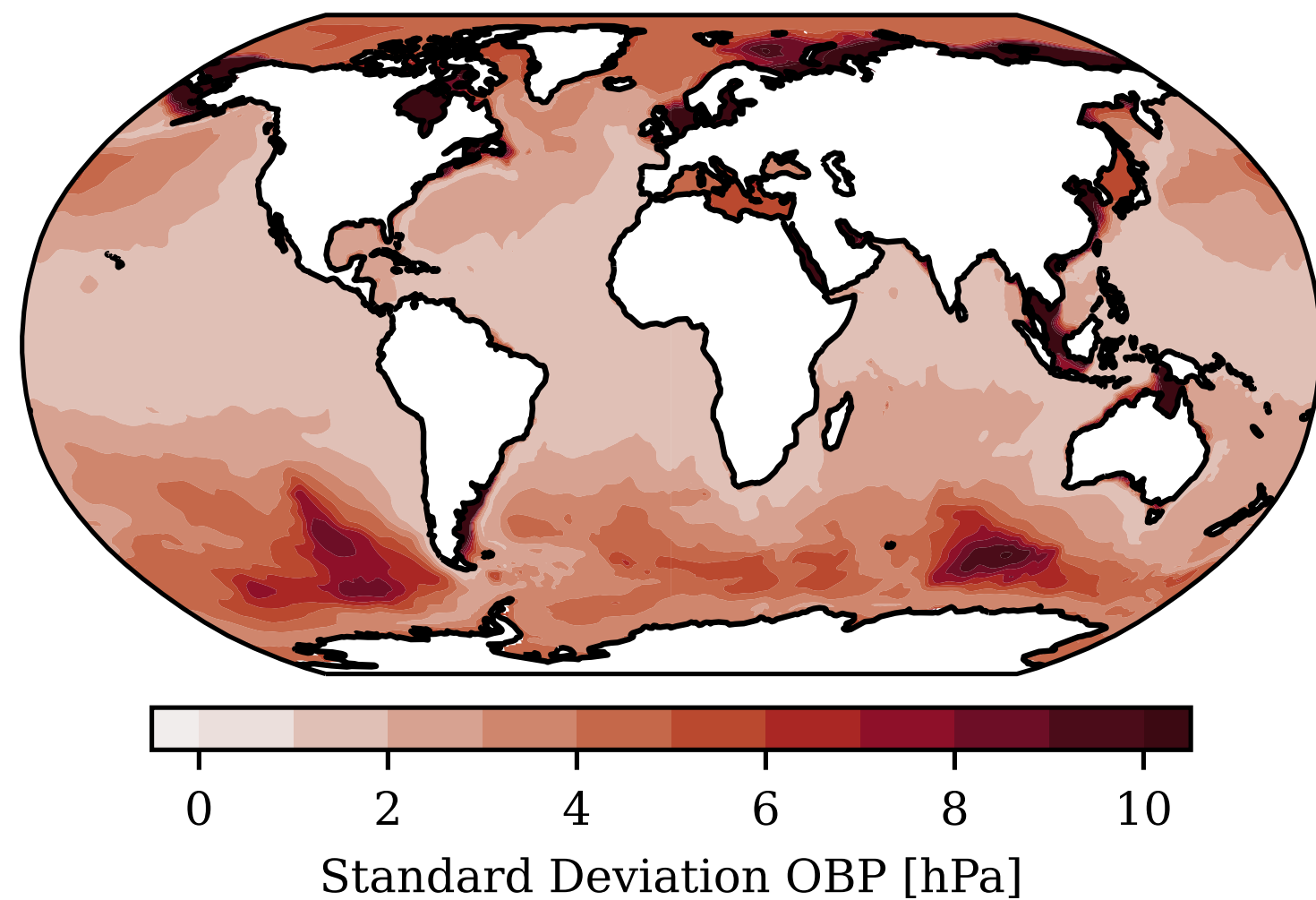
0 40 80 120 160 S_2 [Pa]



0 2 4 6 8 M_2 [Pa] [11]

AOD1B Components: Ocean Bottom Pressure

- No 'operational weather prediction' for ocean bottom pressure (OBP)
- Perform dedicated simulations using ocean general circulation model MPIOM
- Focus on high-frequency variability driven by atmospheric winds and pressure
- MPIOM forced using atmospheric fields from ECMWFs ERA5 & IFS
- No luni-solar tidal forcing but atmospheric induced tides are removed

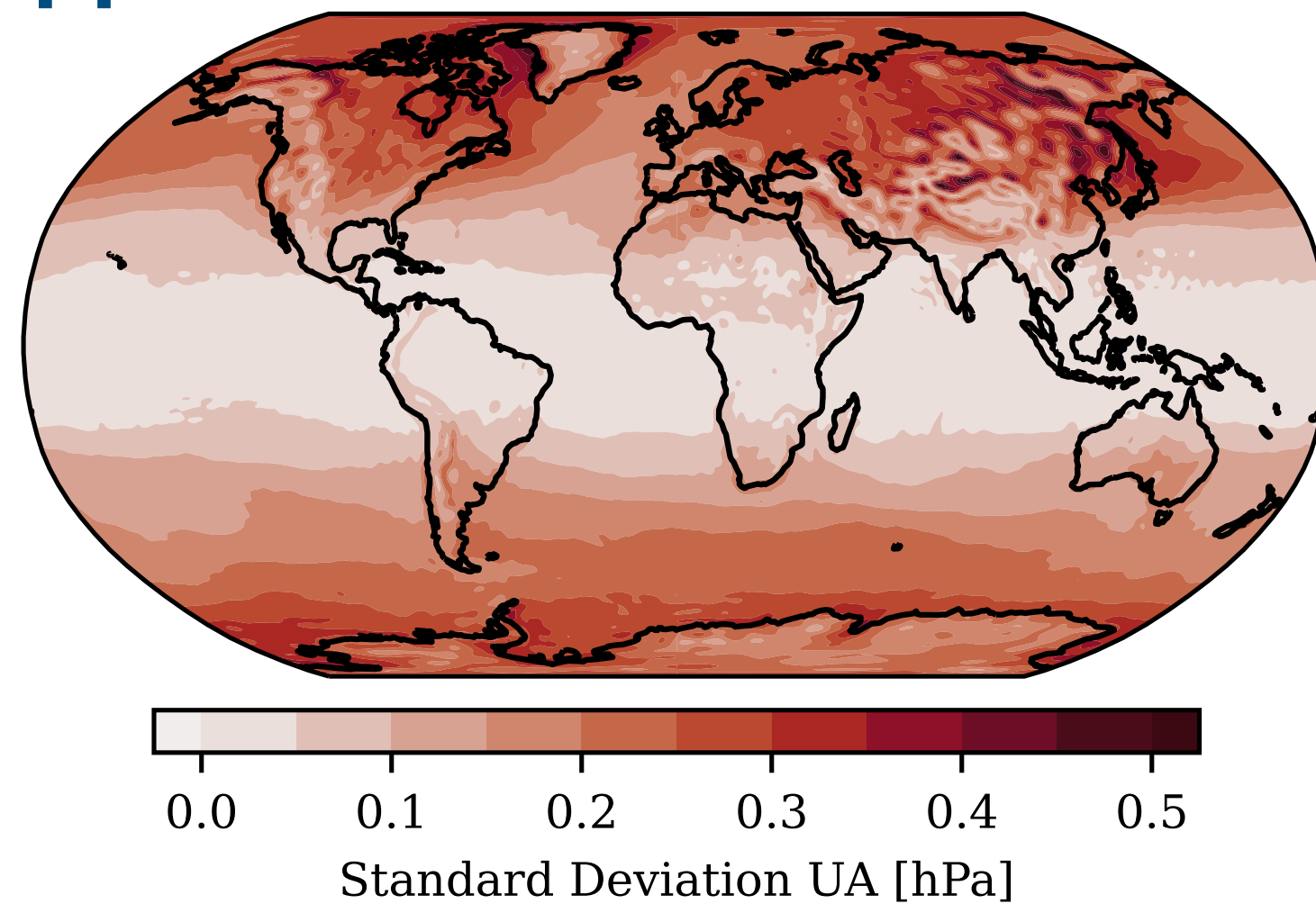


- No data assimilation
- Large-scale variability well represented
- Semi-enclosed seas and some coastal regions are harder to simulate

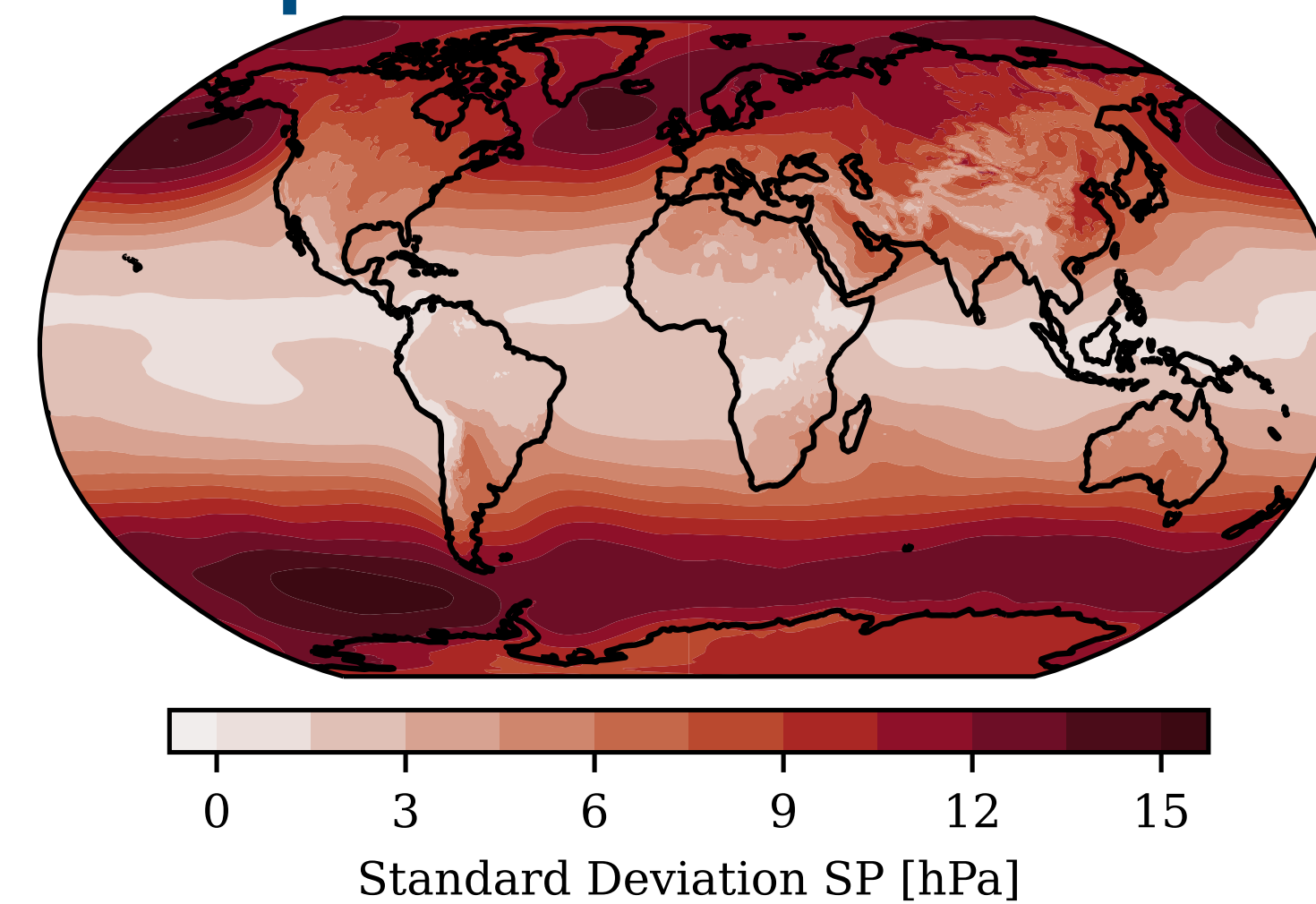
AOD1B Components: Upper Air Variations

- Additional atmospheric contribution: density anomalies in the upper atmosphere
- Common assumption: atmospheric effects confined to very thin layer
- Vertical structure *does* have a small effect
- Calculate contribution via a vertical integration considering temperature and humidity
- Variability small (<0.5 cm EWH) compared to surface pressure

Upper Air Variations:



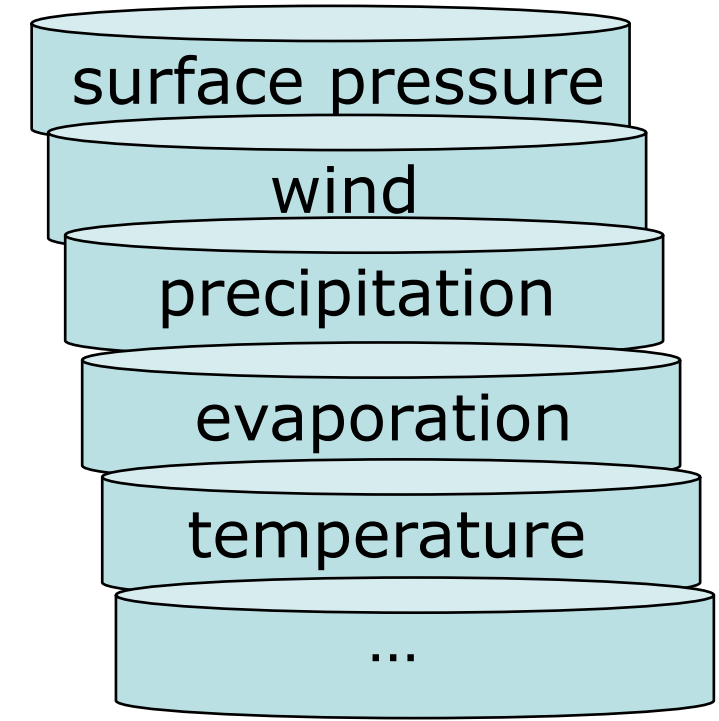
Atmospheric Surface Pressure:



AOD1B Processing at GFZ



latest operational analysis data

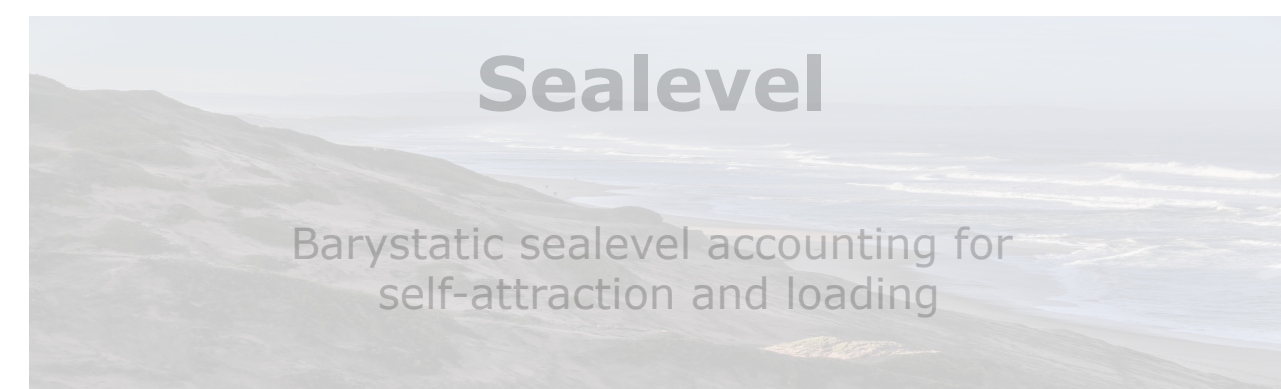
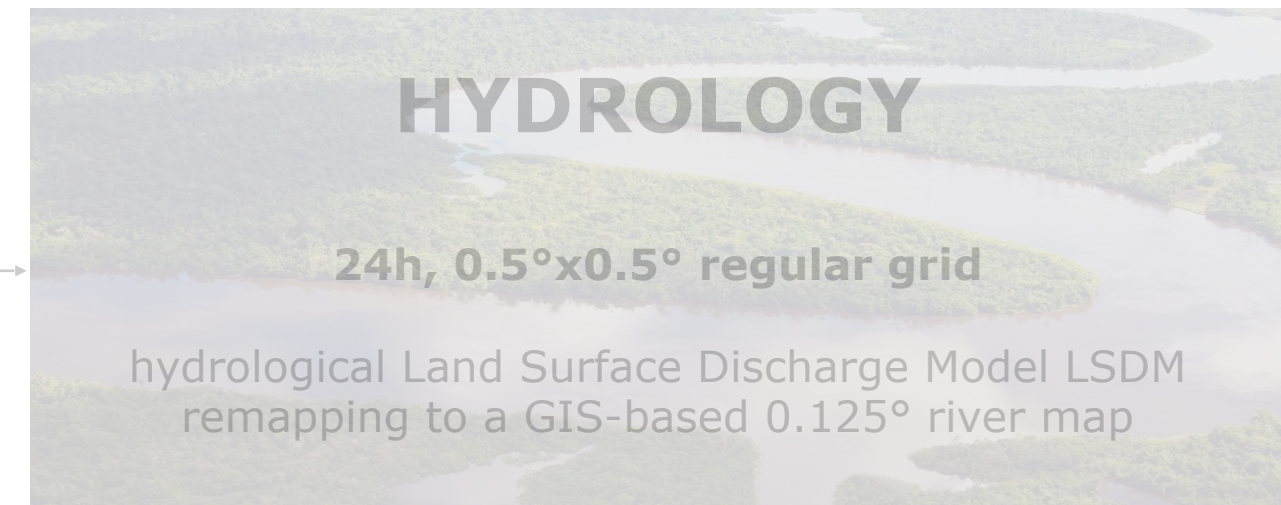
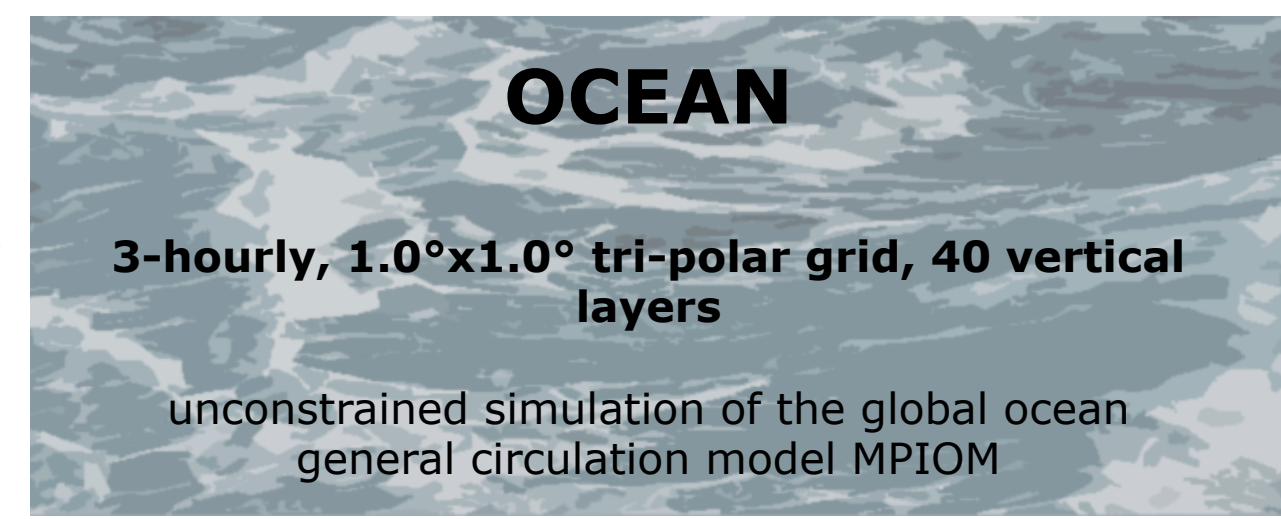
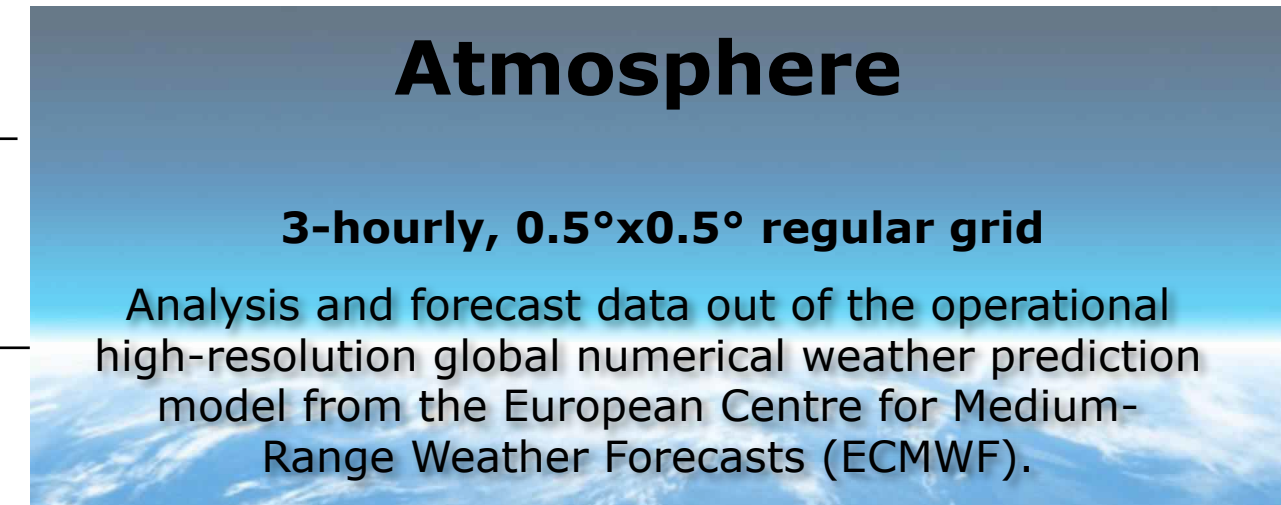
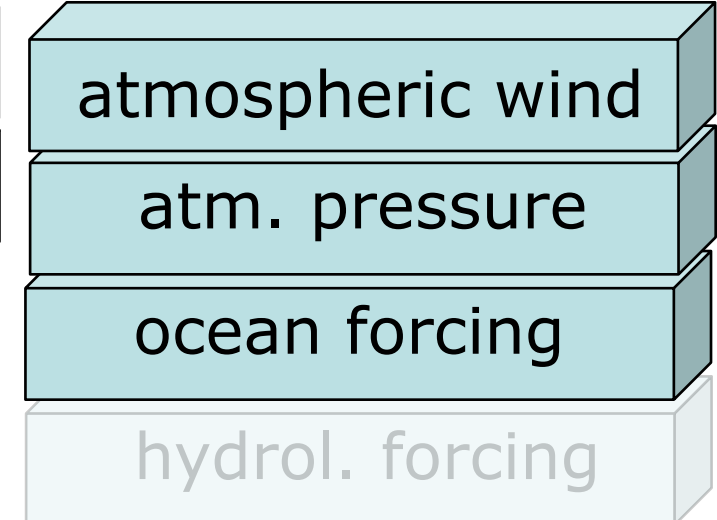
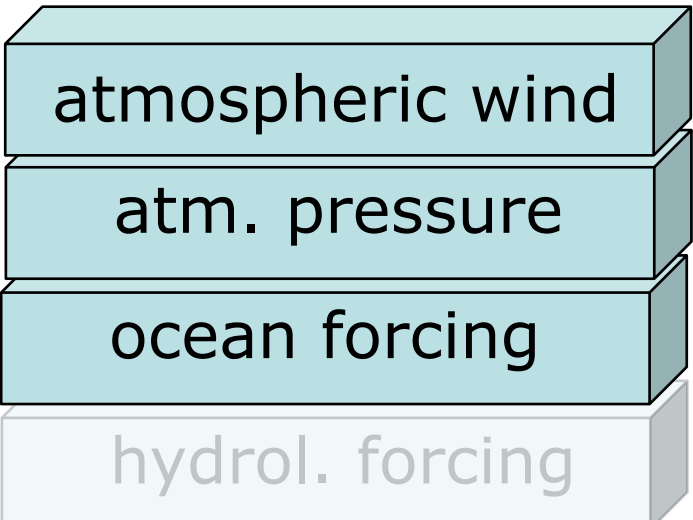


+ 3h forecasts

6-day forecasts

last day 3-hourly

forecast 3-hourly



wind

mass

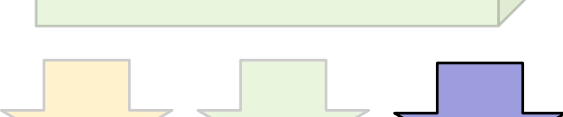
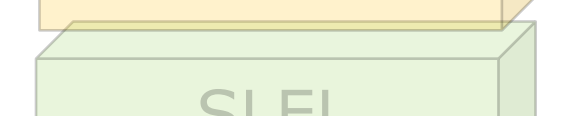
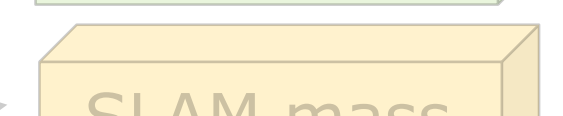
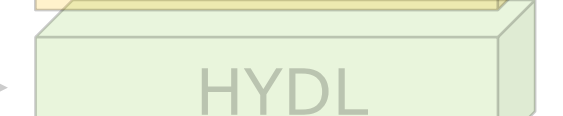
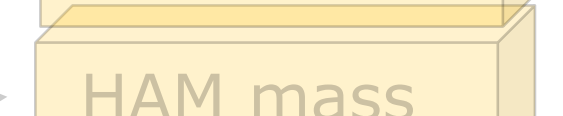
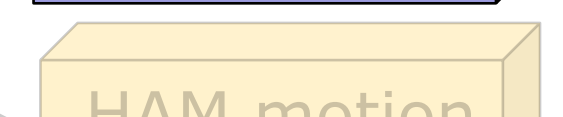
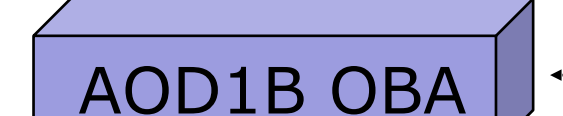
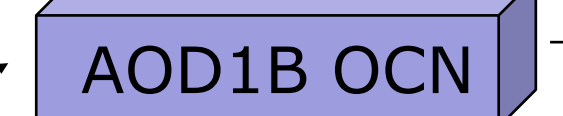
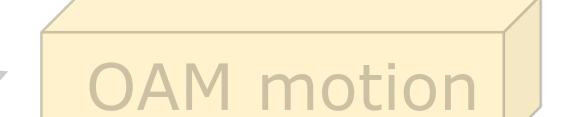
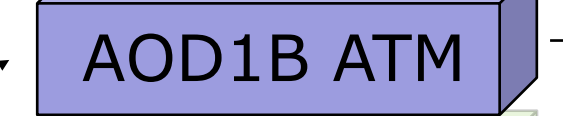
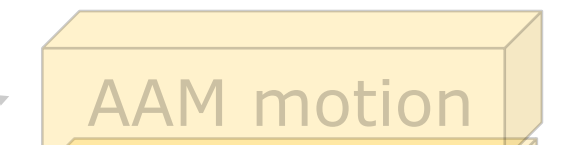
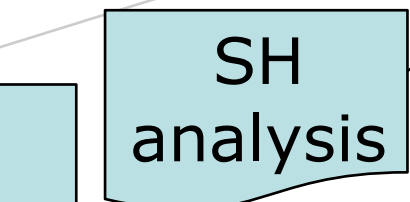
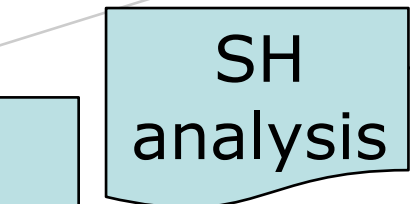
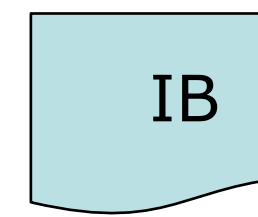
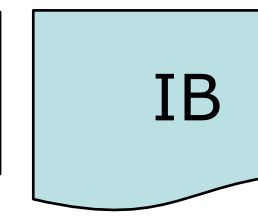
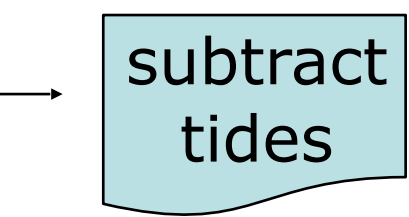
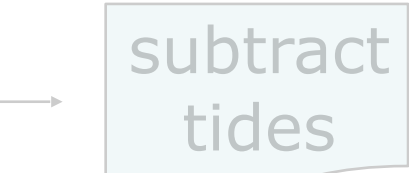
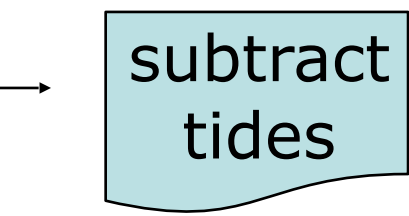
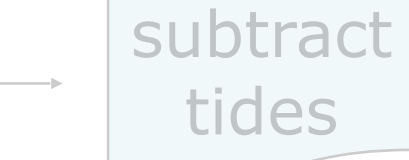
current

mass

flow

mass

sealevel equation



Some Technical Details

- AOD1B is provided as fully normalised Stokes coefficients up to d/o 180
- 3-hourly resolution
- Operationally updated daily
- 4 sets of coefficients:
 - ATM: surface pressure & upper air variability over continents
 - OCN: dynamic ocean bottom pressure
 - **GLO**: combination of ATM + OCN
 - OBA: same as GLO but without upper air contribution
- Monthly coefficients are available to be added back to the GSM fields for specific application (GAA, GAB, GAC, GAD)
 - Example: applications deriving OBP from GRACE data should restore GAD
- Details in Product Description Document in ISDC

used in GRACE processing



GRACE 327-750
Gravity Recovery and Climate Experiment
Product Description Document
for AOD1B Release 07
(Rev. 7.0, October 13, 2023)

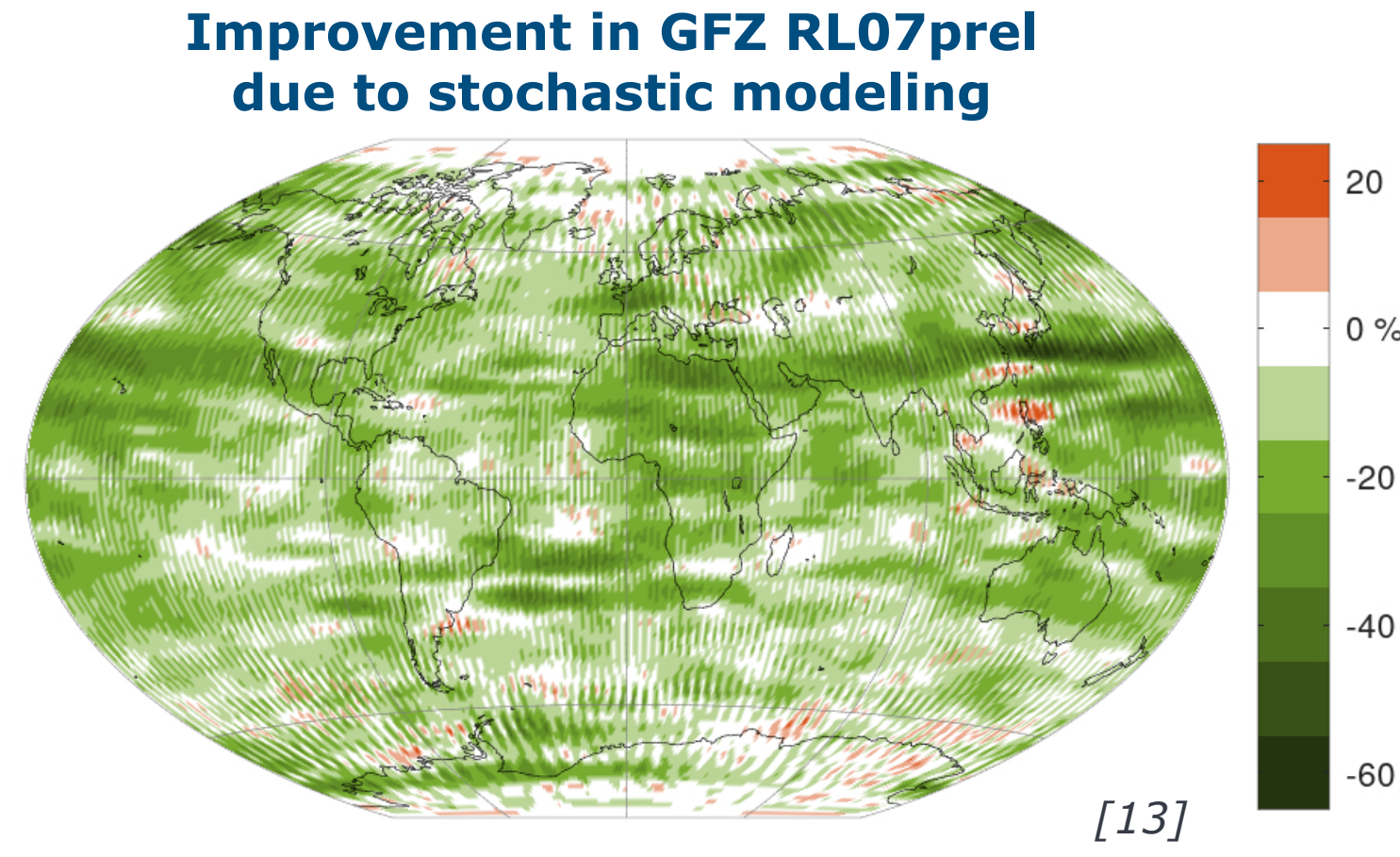
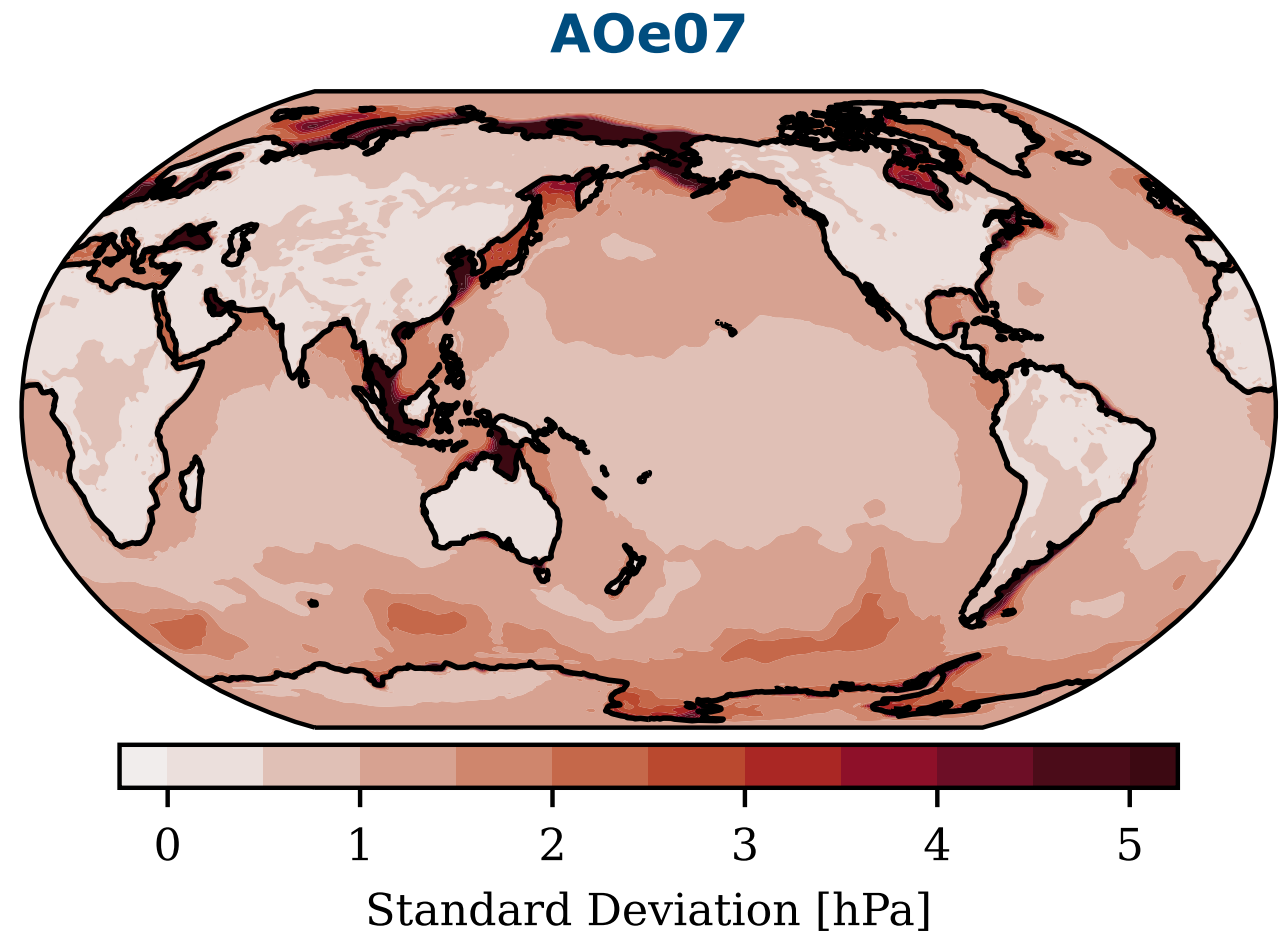
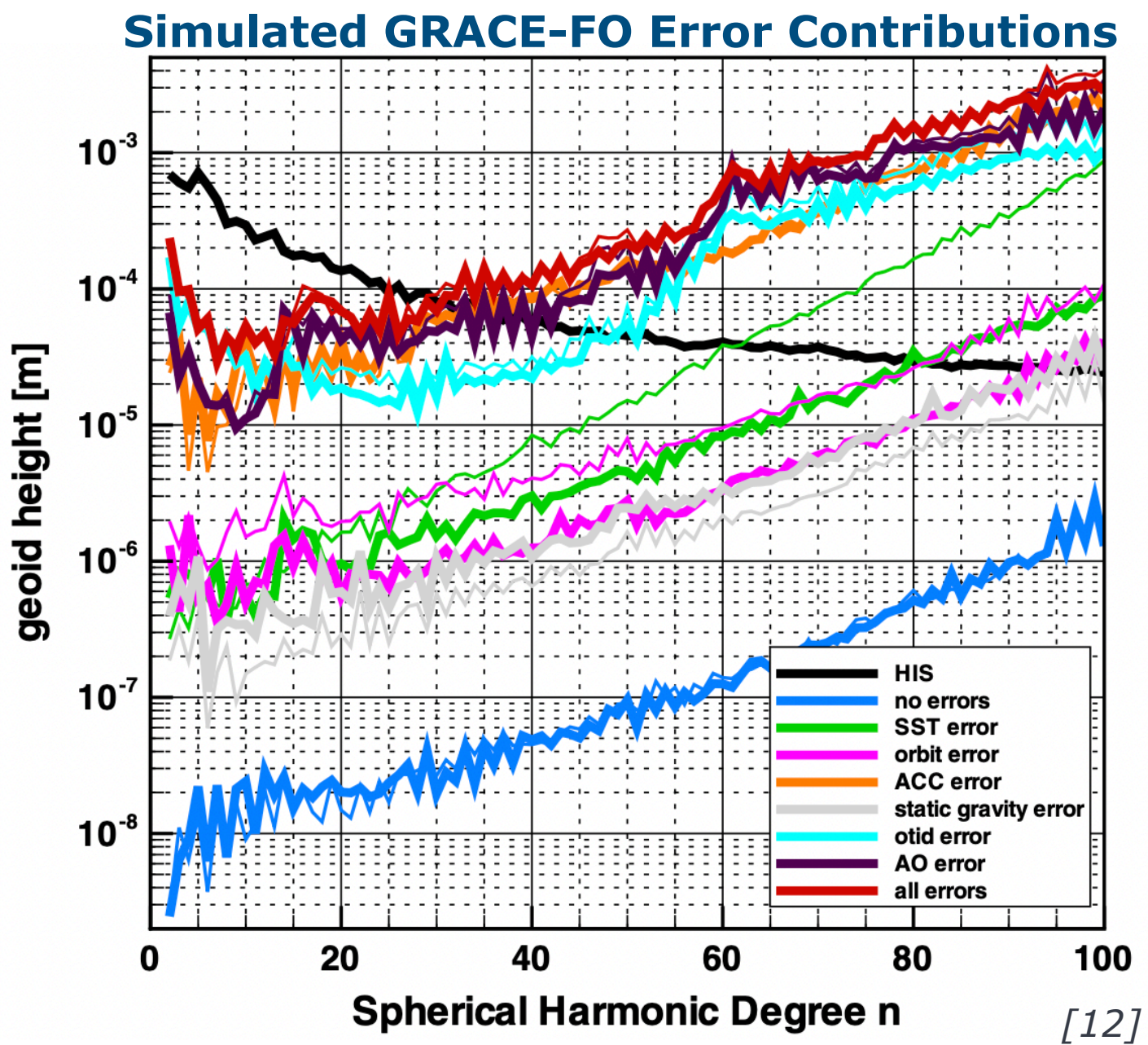
Linus Shihora, Kyriakos Balidakis, Robert Dill,
Henryk Dobslaw
GFZ German Research Centre for Geosciences
Department 1: Geodesy

Residual Errors in Background Models

- Background models are inevitably imperfect
- Residual errors contribute to the overall error of monthly solutions
- Stochastic information on the residual error can help improve gravity fields

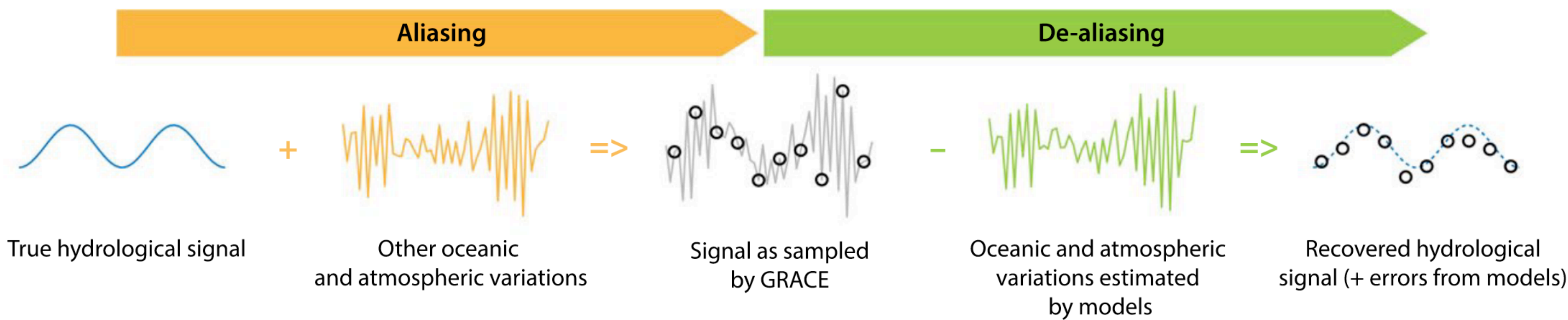
- AOe07: time-series of background model uncertainties
- Based on model differences using ERA5 and MERRA2 re-analyses and simulated OBP

- Weighting observations based on background model accuracy can improve gravity fields up to O(30%)



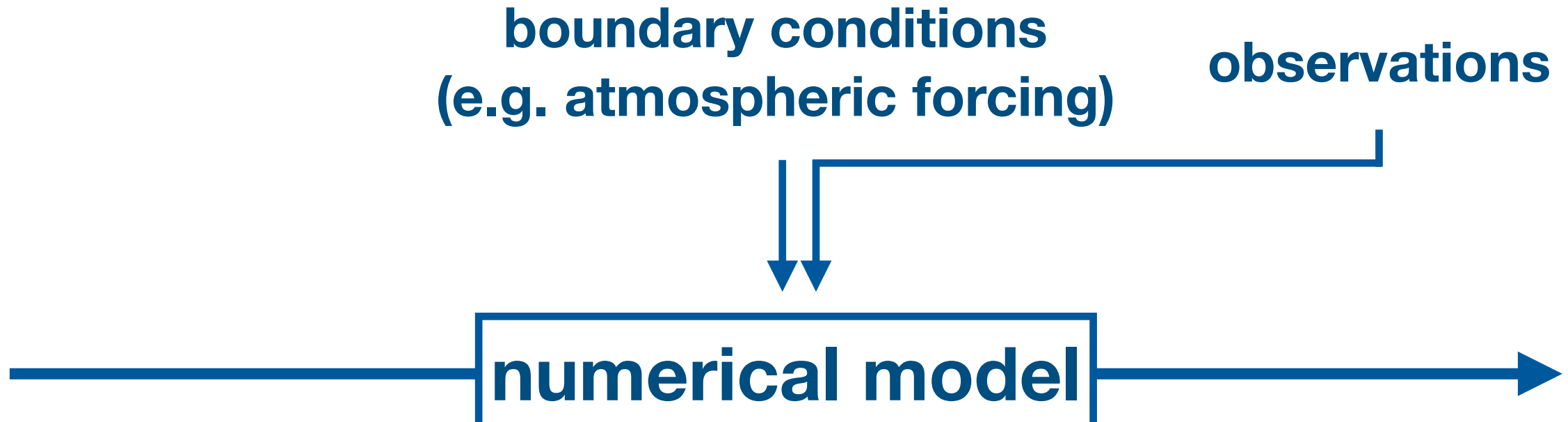
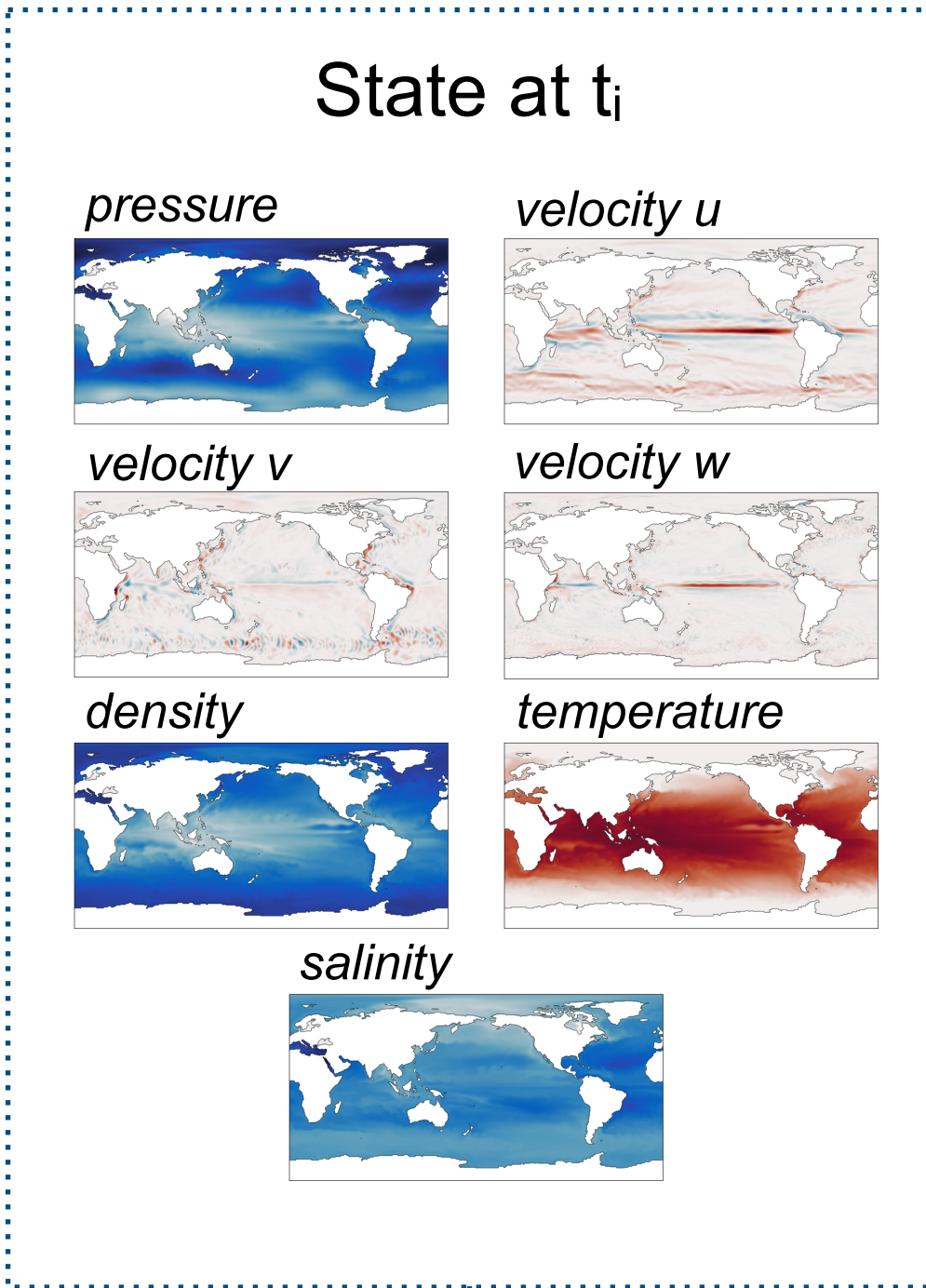
Wrap-Up in Figures

Wrap-Up in Figures



[2]

Wrap-Up in Figures



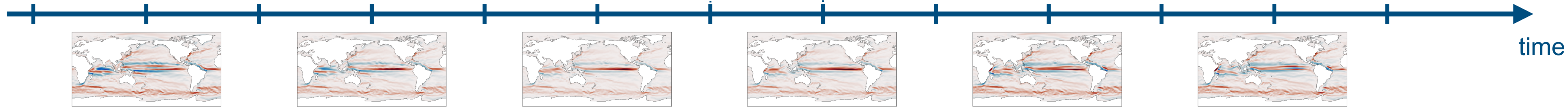
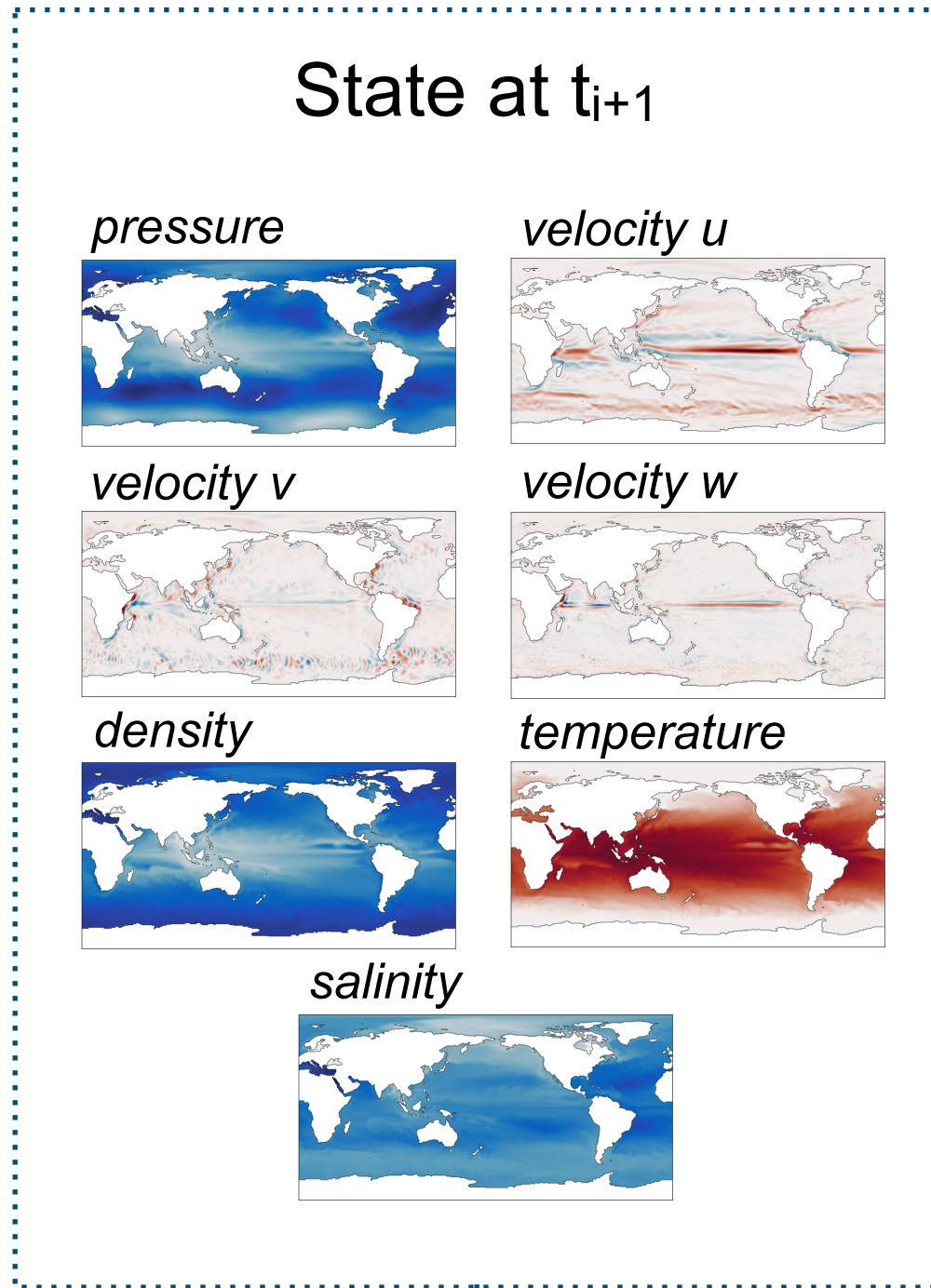
$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0 \quad \rho \left[\frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} \right] = -\nabla p + \rho \nabla \phi + \vec{F}$$

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S + \dot{S}' \quad \rho = \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)]$$

$$C_v \frac{DT}{Dt} = \left[\frac{k}{\rho} \nabla^2 T + Q' \right] - \left[p \frac{D}{Dt} \frac{1}{\rho} \right]$$

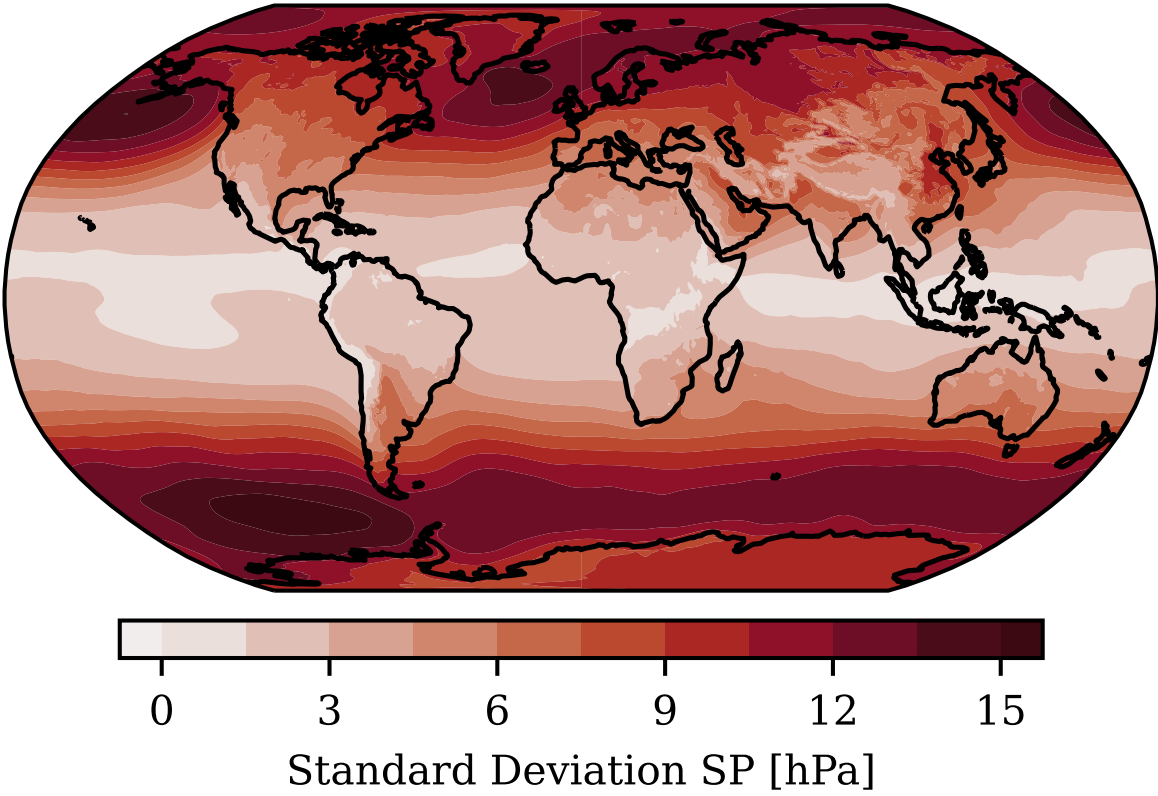
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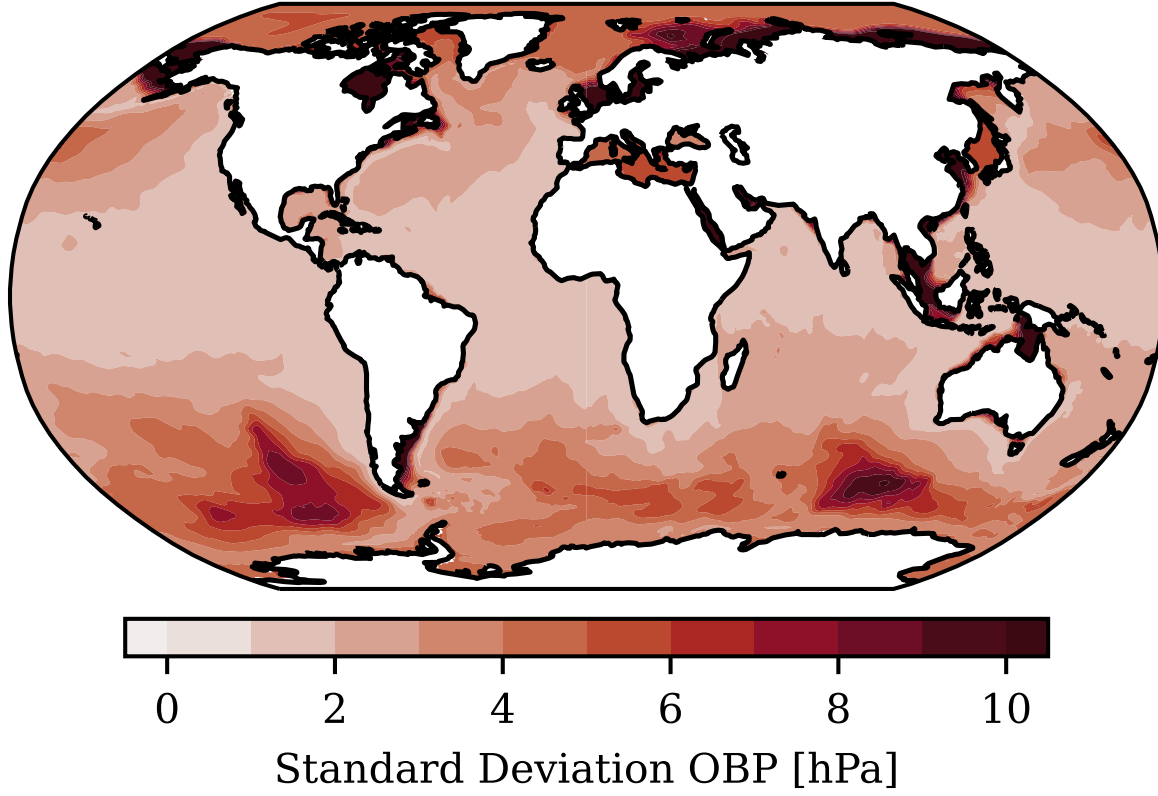
Wrap-Up in Figures

Atmospheric Surface Pressure:



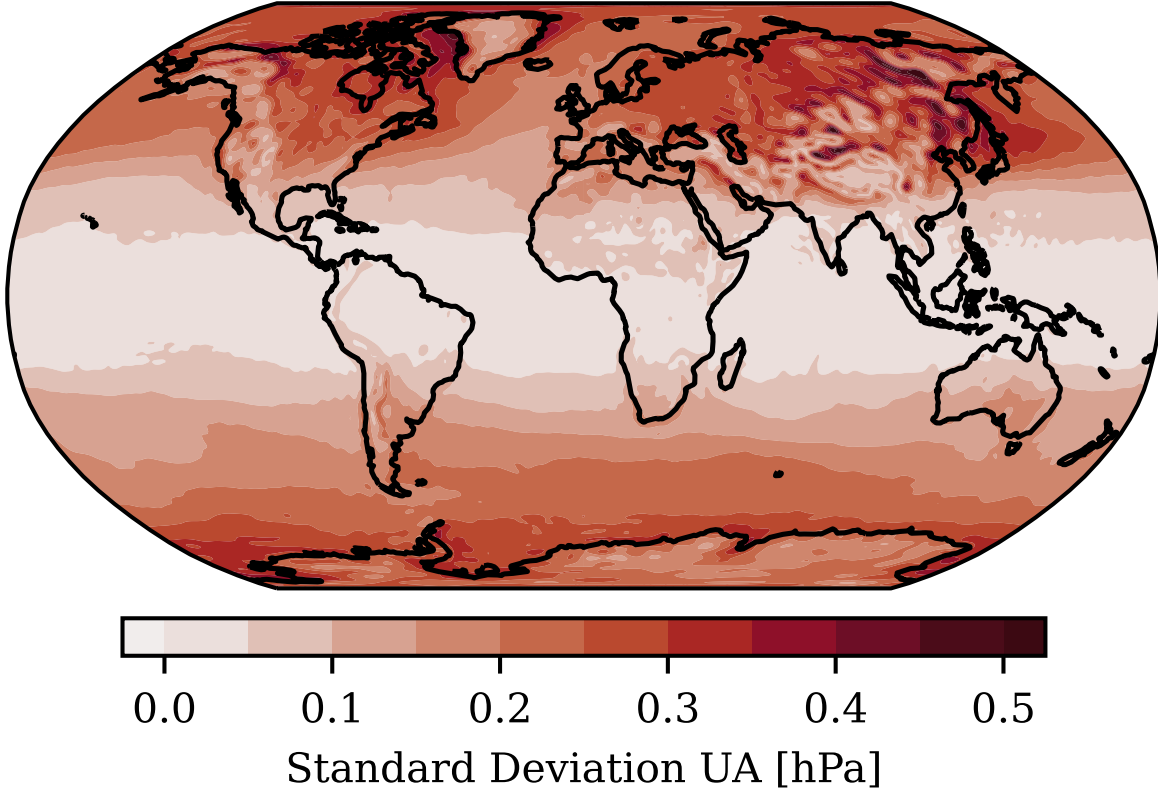
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Ocean Bottom Pressure:



+

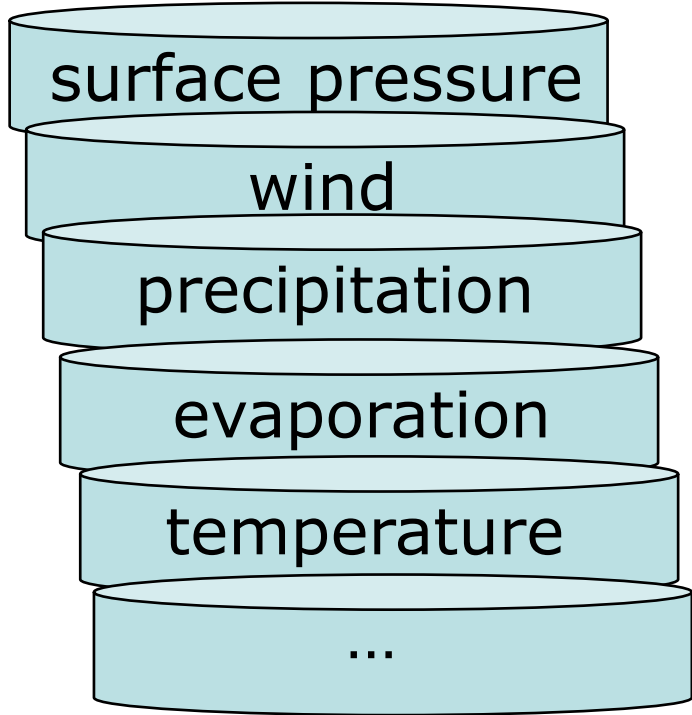
Upper Air Variations:



Wrap-Up in Figures



latest operational analysis data

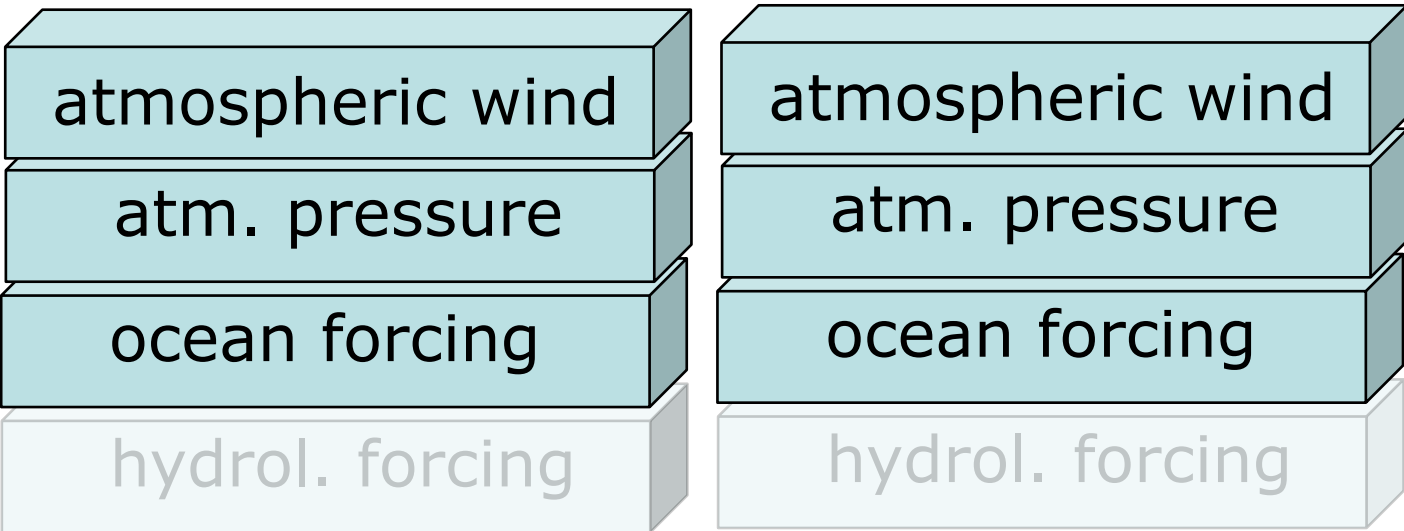


+ 3h forecasts

6-day forecasts

last day
3-hourly

forecast
3-hourly



Atmosphere

3-hourly, 0.5°x0.5° regular grid

Analysis and forecast data out of the operational high-resolution global numerical weather prediction model from the European Centre for Medium-Range Weather Forecasts (ECMWF).

OCEAN

3-hourly, 1.0°x1.0° tri-polar grid, 40 vertical layers

unconstrained simulation of the global ocean general circulation model MPIOM

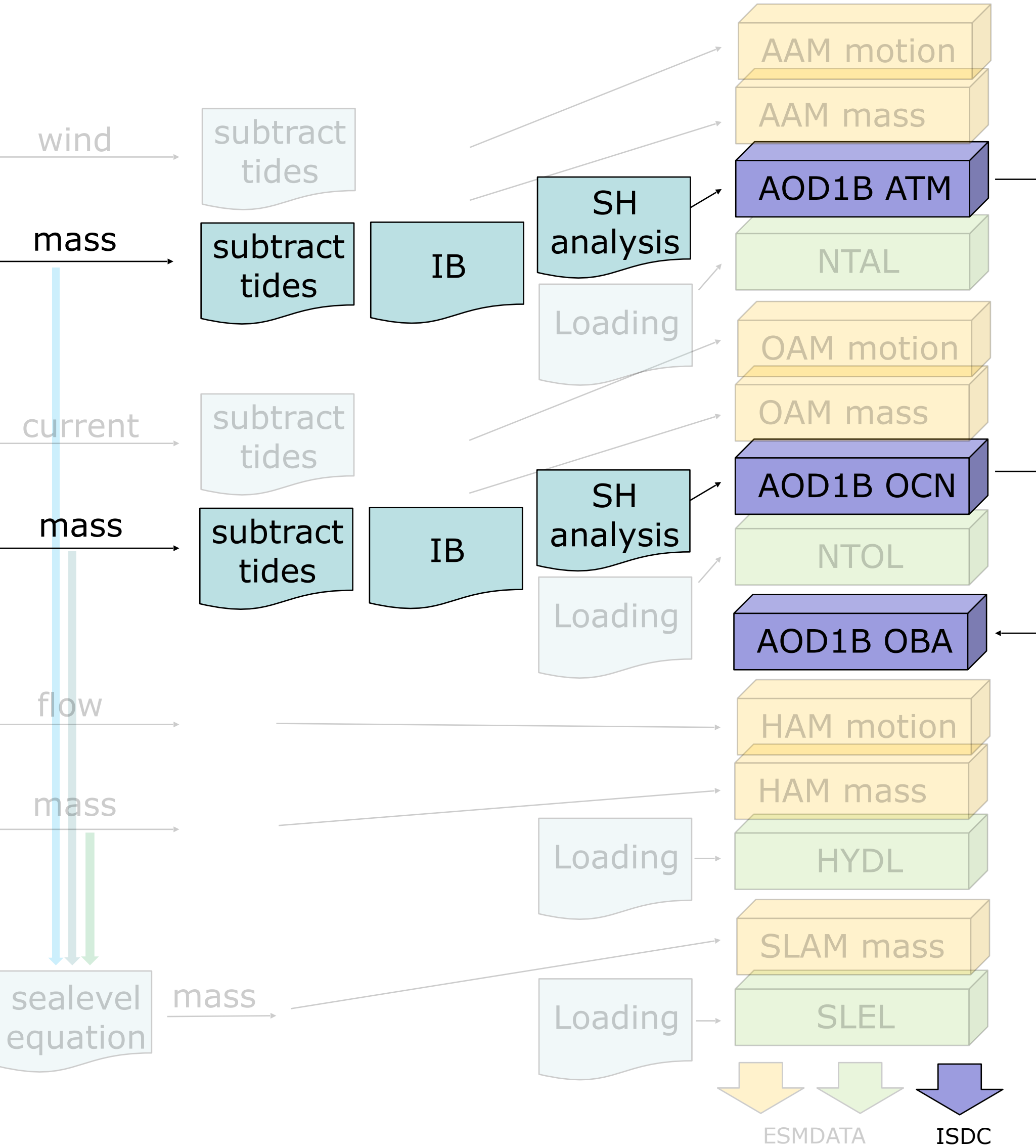
HYDROLOGY

24h, 0.5°x0.5° regular grid

hydrological Land Surface Discharge Model LSDM remapping to a GIS-based 0.125° river map

Sealevel

Barystatic sealevel accounting for self-attraction and loading





NERO GRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

International Spring School
Neustadt an der Weinstraße, Germany, March 10-14, 2025

Background Modelling for Satellite Gravimetry

Linus Shihora (GFZ Helmholtz Centre for Geosciences)

