NEROGRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

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Background Models: Ocean Tides

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What are ocean tides?

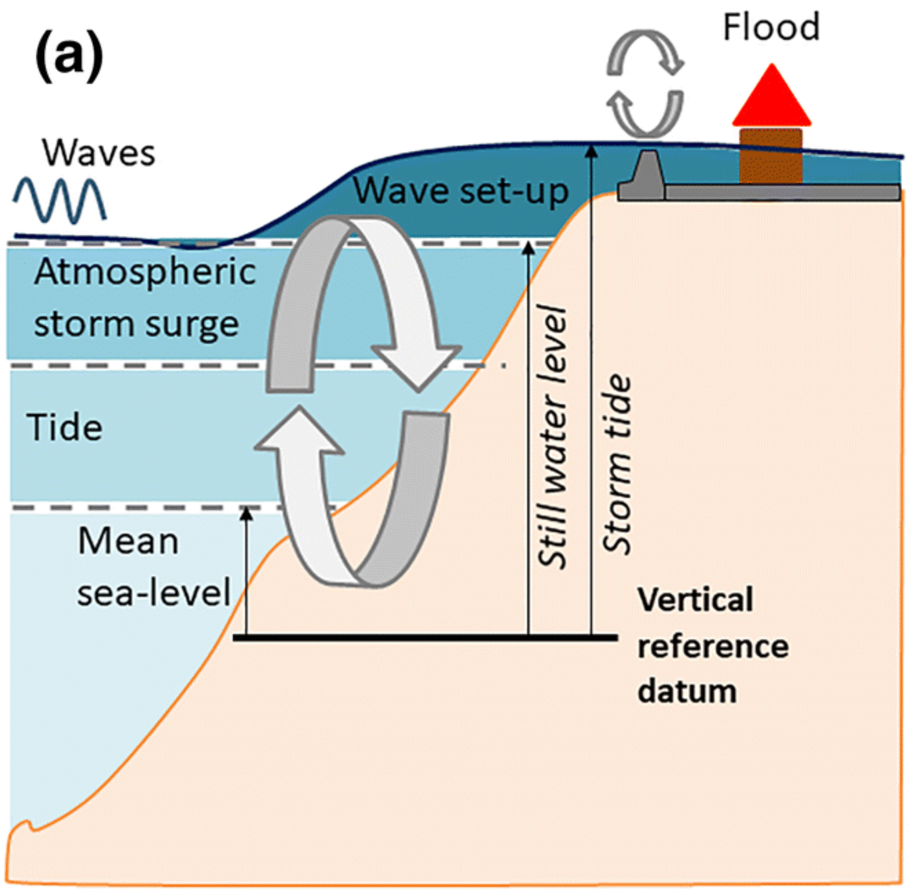
- It is the response of the ocean and the Earth to the gravitational attraction of celestial bodies.
- The two largest responses are caused by the gravitational attraction of the Moon and then the Sun.
- As the position of these bodies vary over time, this results in a variation in the response in both time and space.
- Due to the position of these bodies being very well known, the estimation and prediction of ocean tides can be done.

Why are ocean tides important

- Large societal benefit
  - Fishing Industry
  - Coastal flooding
  - Navigation
- Tidal Energy
- Ocean mixing
- Intertidal zone for marine life
- Sea-level rise research
- **Gravity field retrievals**

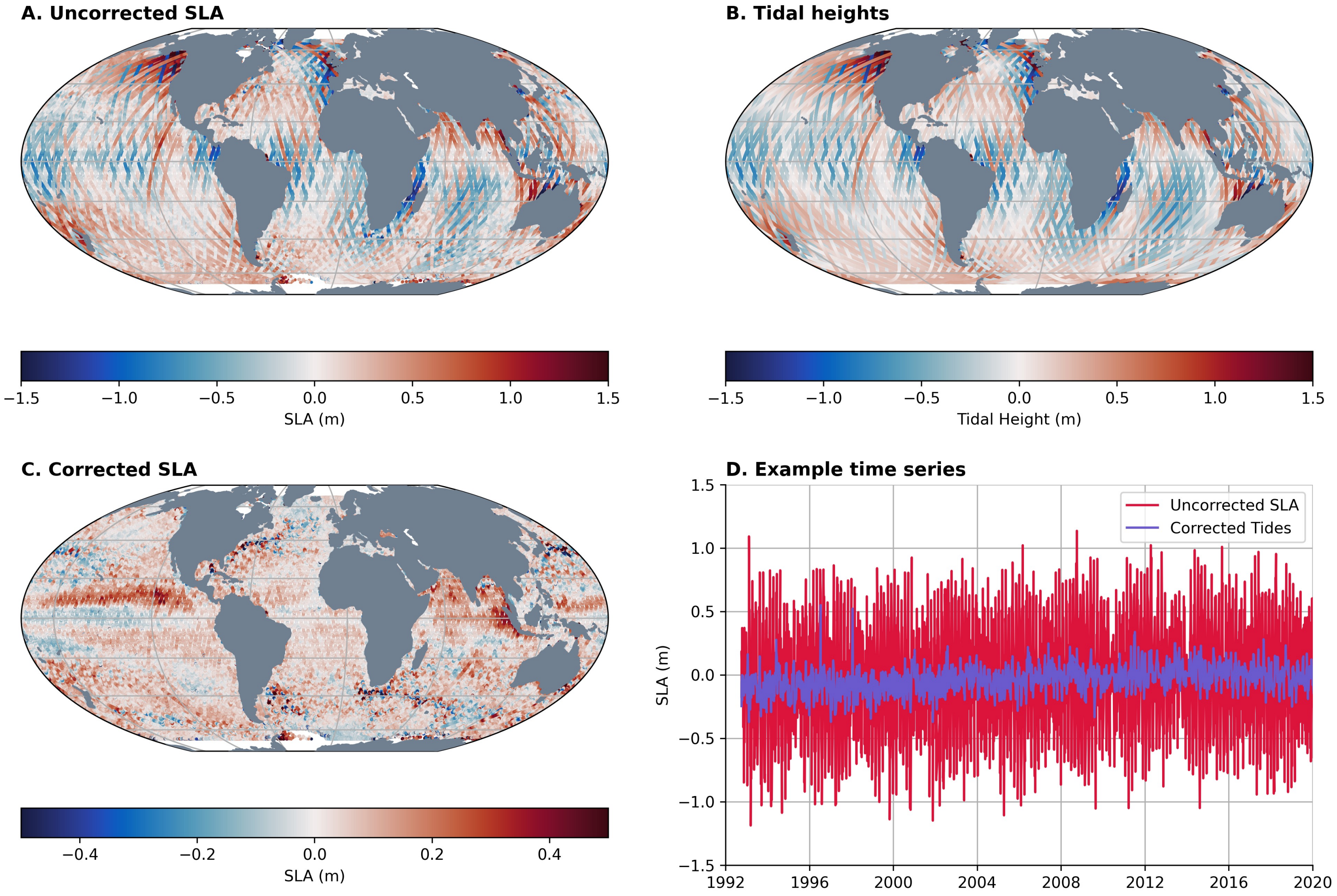
The Intertidal Zone

Tides in the context of studies on sea-level



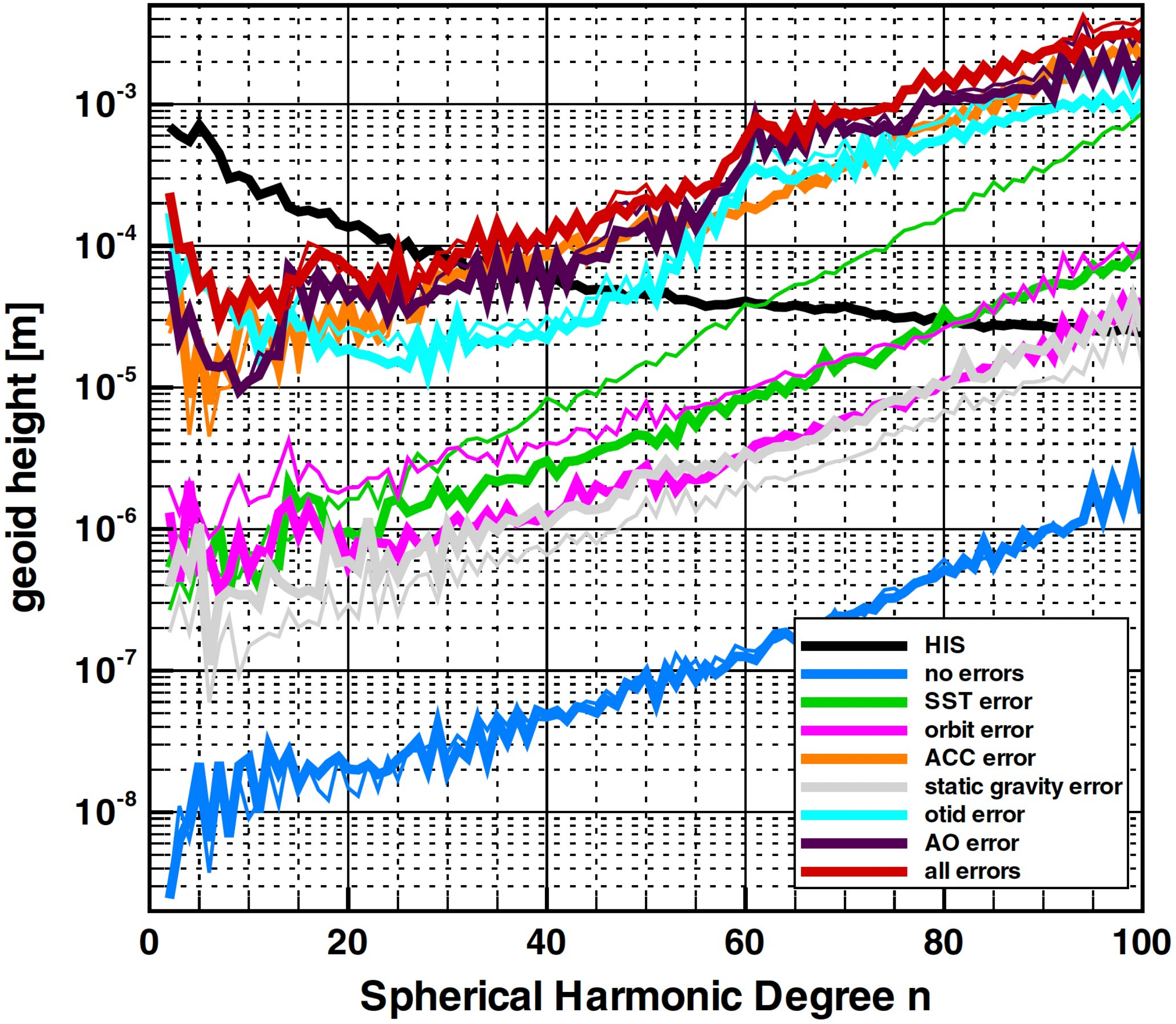
- Estimates of sea-level from, for example, Satellite Altimetry are influenced by the tides.
- The cycle of the tides can result in over or under-estimated sea-level estimations if tides are not taken into account.
- Therefore, ocean tides need to be removed from the signals of satellite altimetry and tide gauges in order to get more accurate estimates of the sea-level changes.

Why we need to correct for tides



Tides in Satellite Gravimetry

- Tides remain one of, if not the greatest error contributions in the GRACE gravity field solutions.
- This is due to aliasing of the tides which influences the retrieval of data from GRACE.
- Significant efforts are on-going to resolve this but it is not a simple solution.



Tides in Satellite Gravimetry

Ocean tides == short-time mass redistribution

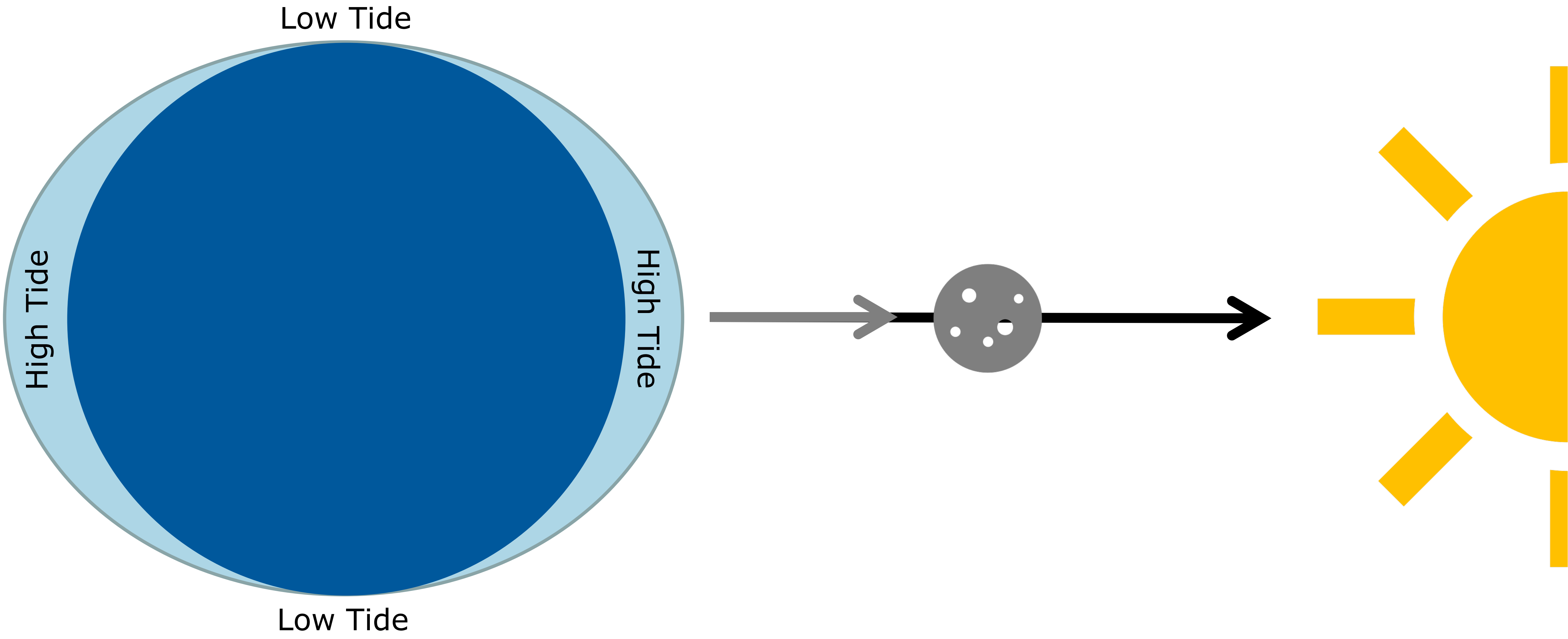
⇒ effect of ocean tides has to be removed from gravity observations in order to obtain models describing exclusively the parts of Earth's gravity field we are interested in and to account for inability of GRACE to resolve higher temporal frequencies

⇒ needed: global ocean tide models with high accuracy and reliable uncertainties

- **Imperfect ocean tide models are considered among the most limiting factors in determining high-resolution temporal gravity fields.**
- **Only incomplete information about the uncertainties of current ocean tide models is available => tidal models are often assumed to be error-free**

Theory of Ocean Tides

The Basics of Ocean Tides, Equilibrium Tide Theory



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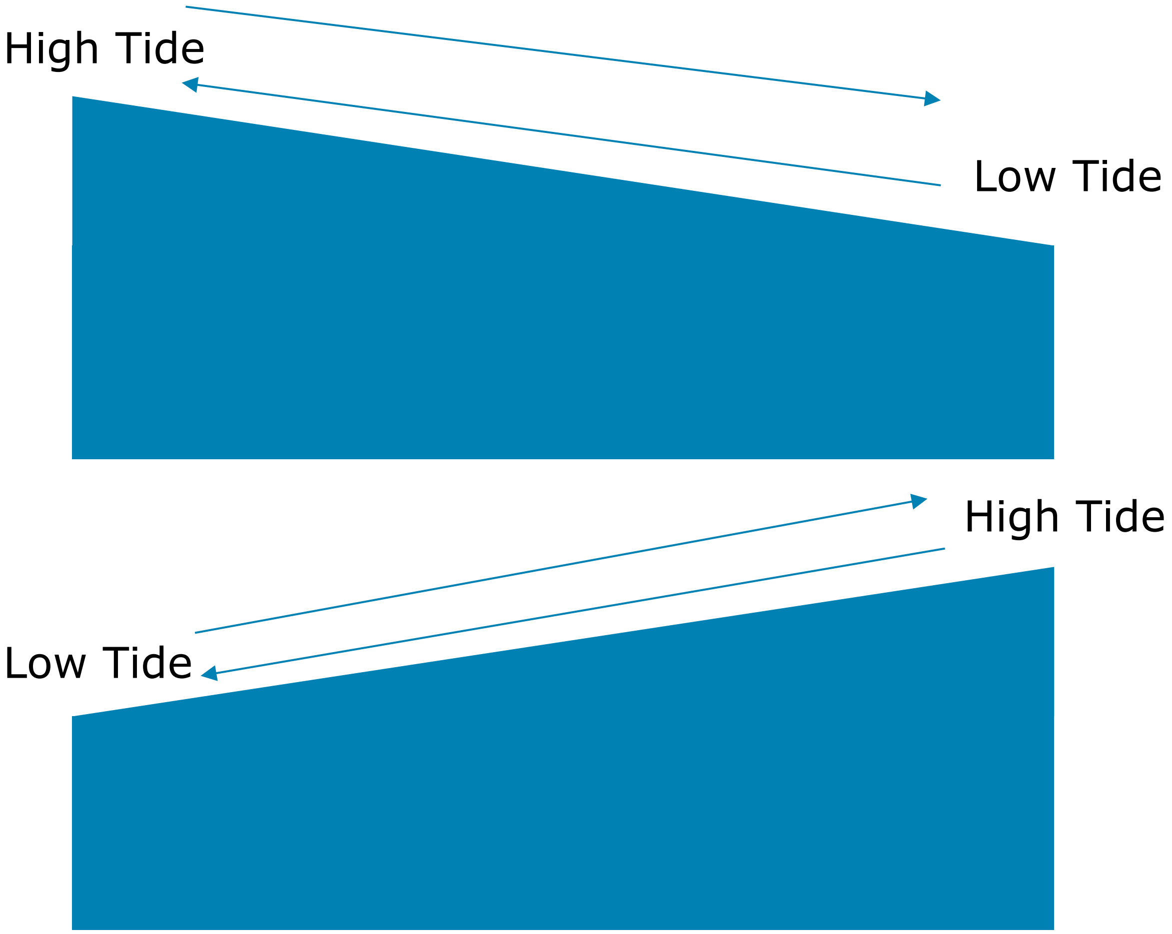
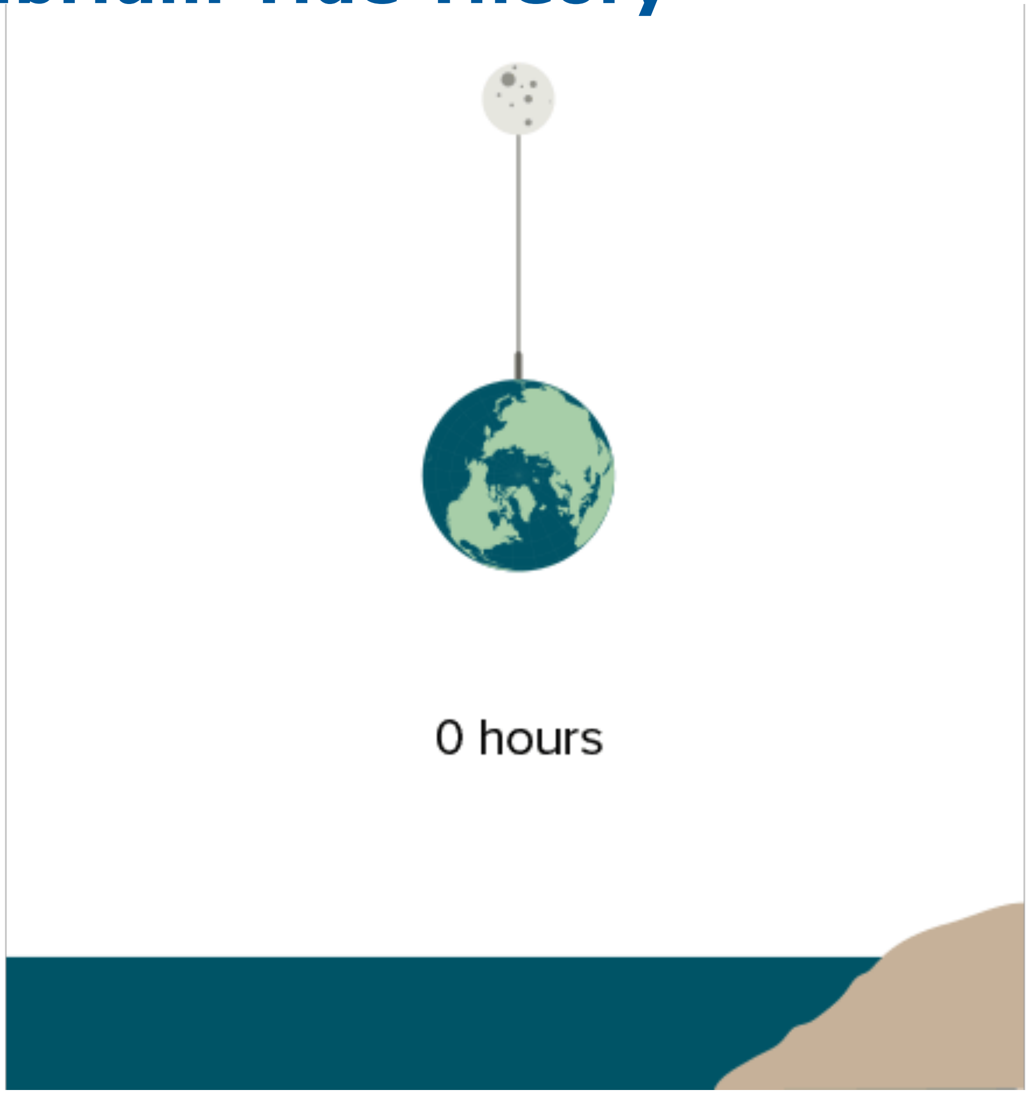
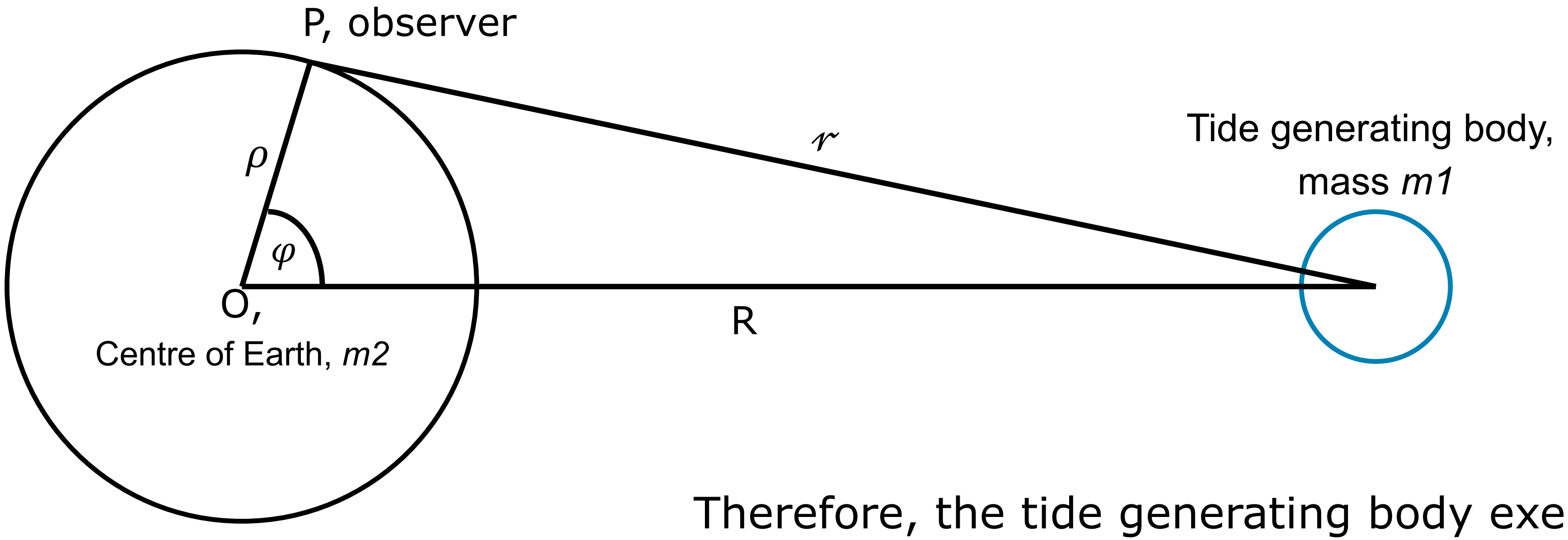




The Tide Generating Potential



Newtons first law of gravitation states:

$$F = \frac{GM}{R^2}$$

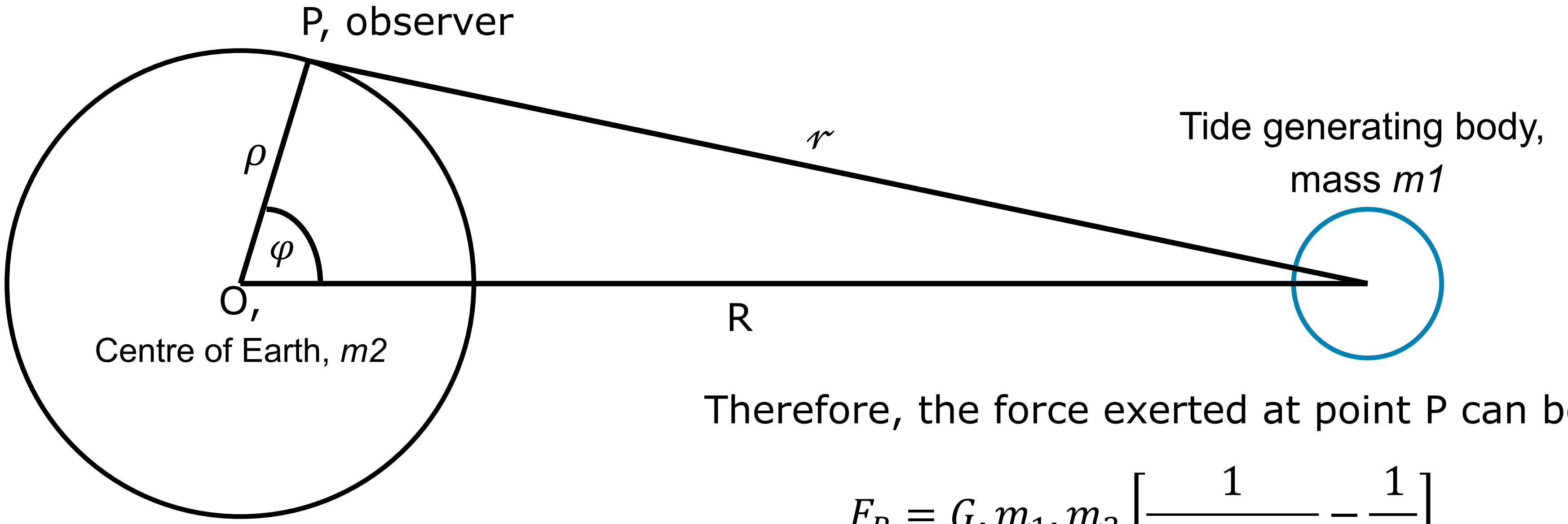
G : the Newtonian Gravitational Constant
= 6.674 x $10^{-11}\ m^3.kg^{-1}.s^{-2}$

Therefore, the tide generating body exerts a force, ***F***, on the centre of Earth:

$$F = G\frac{m1.m2}{R^2}$$

m1: mass of the TGB
m2: mass of the Earth

The Tide Generating Potential



Therefore, the force exerted at point P can be expressed as:

$$F_P = G.m_1.m_2\left[\frac{1}{(R-\rho\ )^2}-\frac{1}{R^2}\right]$$

$$F_P = G.\frac{m_1.m_2}{R^2}\left[\frac{1}{\frac{1}{R^2}(R-\rho\ )^2}-1\right]$$

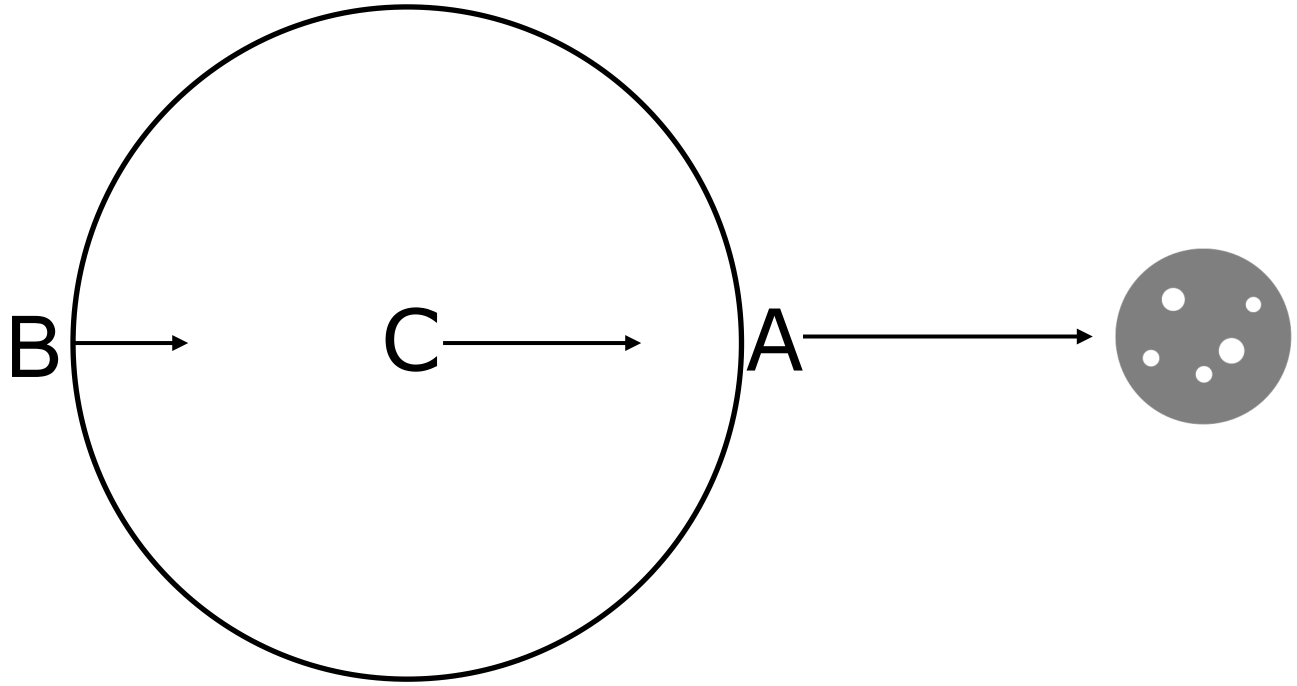
$$F_P = G.\frac{m_1.m_2}{R^2}\left[\frac{1}{(1-\frac{\rho}{R^2})^2}-1\right]$$

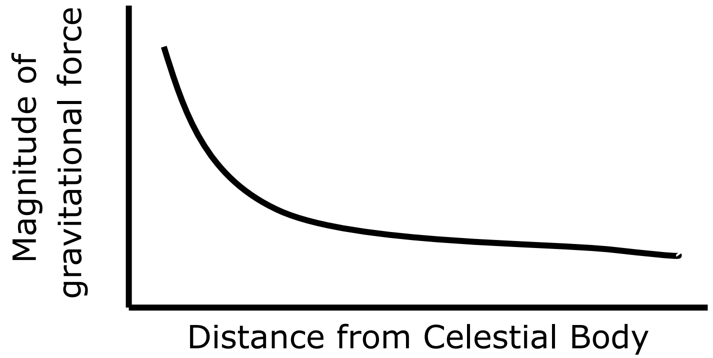
The tide generating force is considered a differential force, as it corresponds to the difference exerted at a certain point on the Earth's surface (say P) relative to the force exerted at the center of the Earth.

Question!

Why does the tide bulge on the other side of the Earth?

Why does the tide bulge on the other side of the Earth?



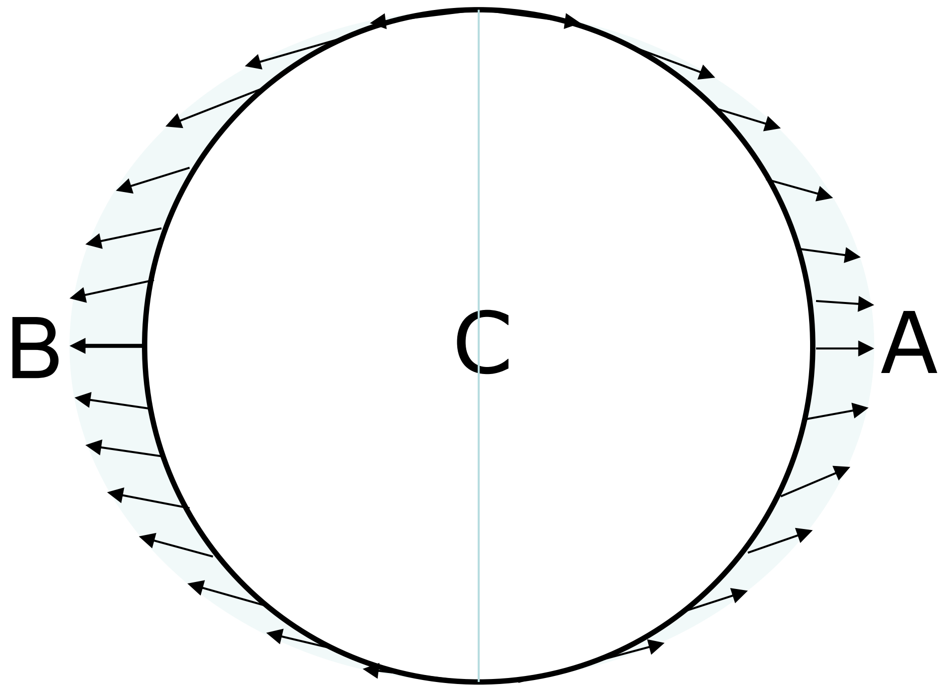


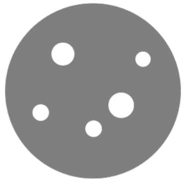
$$F_A > F_C, +F_A$$
$$F_B < F_C, -F_B$$

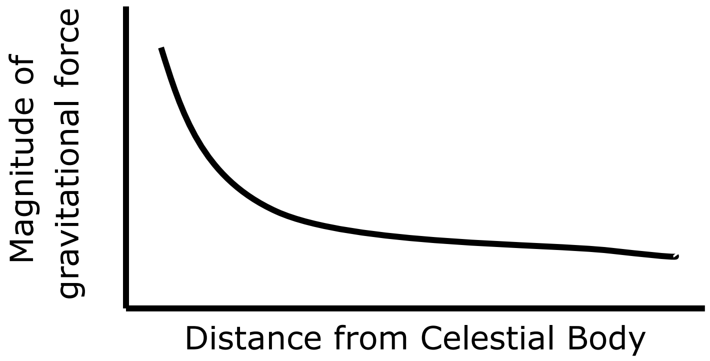
E.g. F_A = 1.1 N
F_C = 1.0 N
F_B = 0.9 N

Therefore, net:
F_A = +0.1 N
F_B = - 0.1 N

Why does the tide bulge on the other side of the Earth?







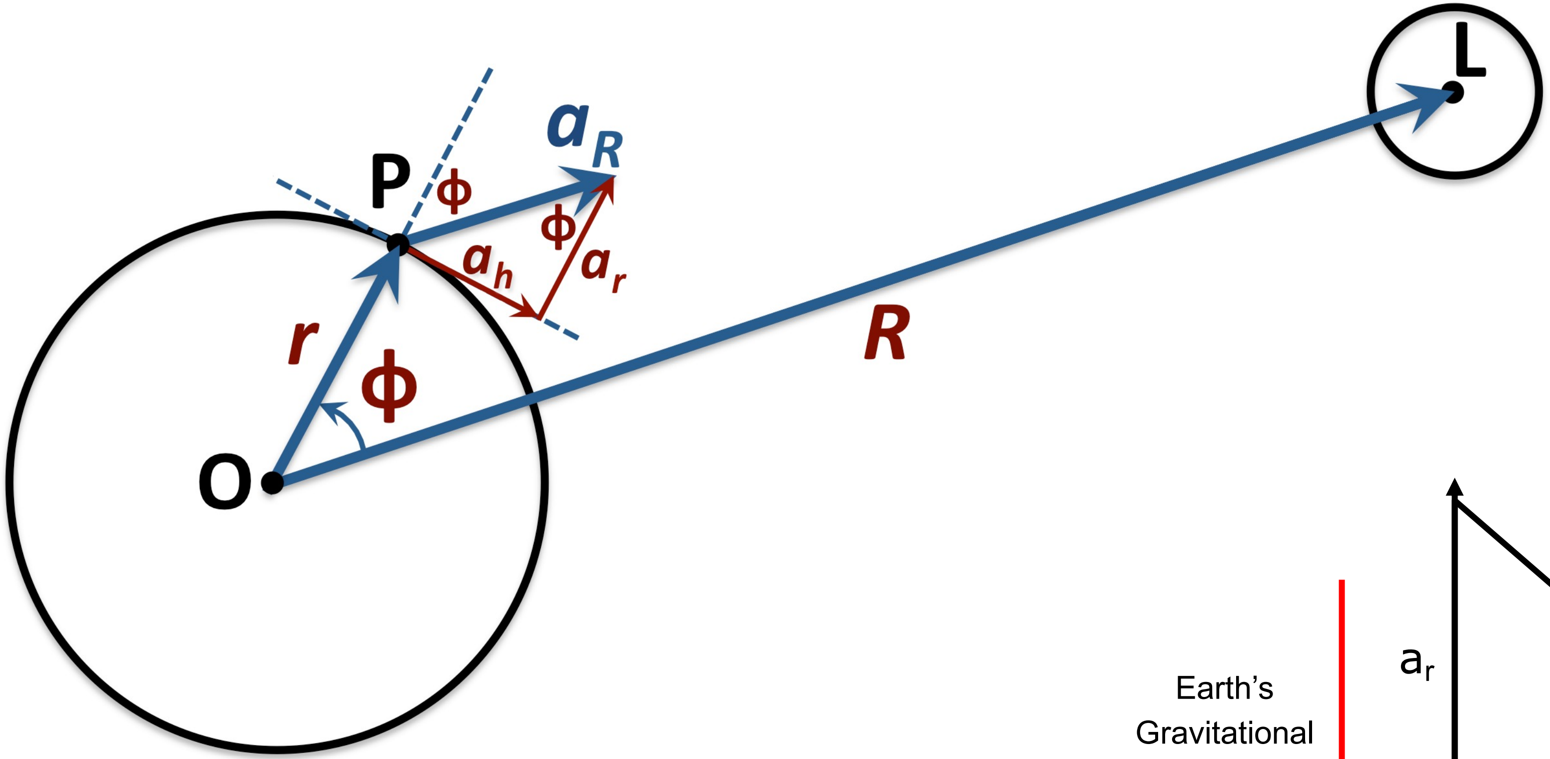
$$F_A > F_C, +F_A$$
$$F_B < F_C, -F_B$$

E.g. F_A = 1.1 N
F_C = 1.0 N
F_B = 0.9 N

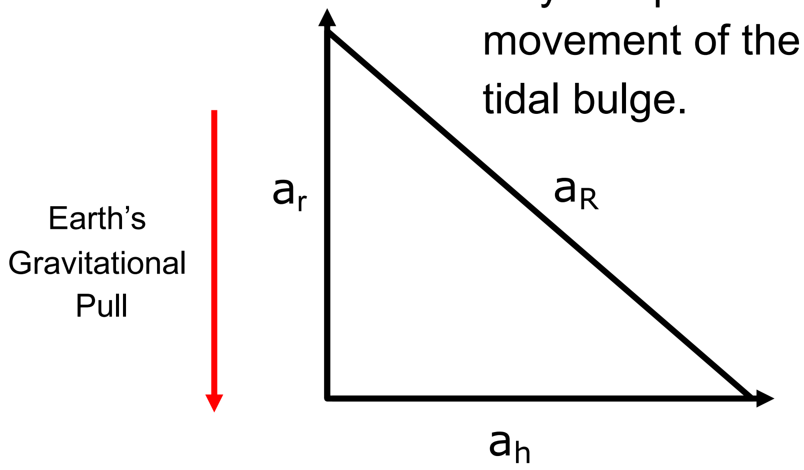
Therefore, net:
F_A = +0.1 N (towards body)
F_B = - 0.1 N (away from body)

Question! Are tides caused by the horizontal (tangential) or vertical (radial) component of the tidal force?

Ocean Tides Are Caused by the Tangential Component!



- Gravitational Attraction at point P (a_R), has a tangential (a_h) and radial (a_r). This can be determined by simple trigonometry.
- a_r <<< Earth's Gravity (9.8 m/s), therefore can be neglected.
- Therefore, the tangential component is the only component determining the movement of the ocean and causing the tidal bulge.



Equilibrium Tide

Tidal Species	Name	Equilibrium Amplitude† (m)	Period (hr)
Semidiurnal	$n_1 = 2$		
Principal lunar	M_2	0.242334	12.4206
Principal solar	S_2	0.112841	12.0000
Lunar elliptic	N_2	0.046398	12.6584
Lunisolar	K_2	0.030704	11.9673
Diurnal	$n_1 = 1$		
Lunisolar	K_1	0.141565	23.9344
Principal lunar	O_1	0.100514	25.8194
Principal solar	P_1	0.046843	24.0659
Elliptic lunar	Q_1	0.019256	26.8684
Long Period	$n_1 = 0$		
Fortnightly	Mf	0.041742	327.85
Monthly	Mm	0.022026	661.31
Semiannual	Ssa	0.019446	4383.05

†Amplitudes from Apel (1987)

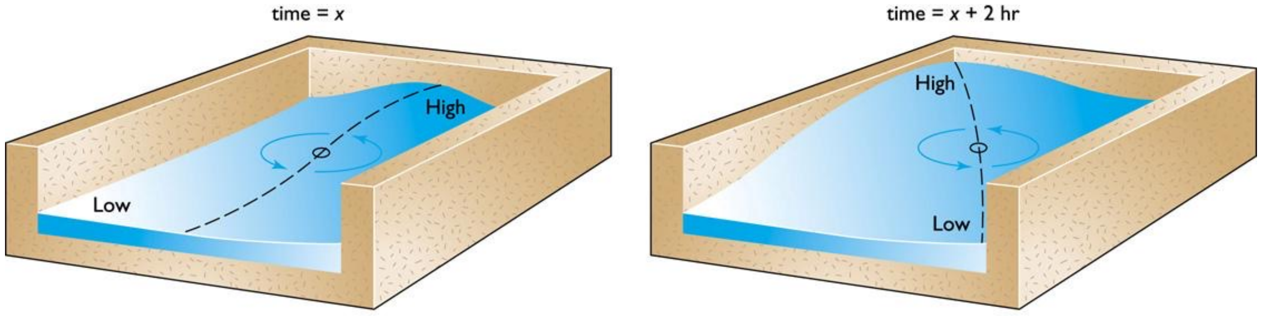
The Equilibrium Tide is defined as the elevation of the sea surface that would be in equilibrium with the tidal forces if the Earth were covered with water to such a depth that the response of the water is instantaneous.

Tides are split into:

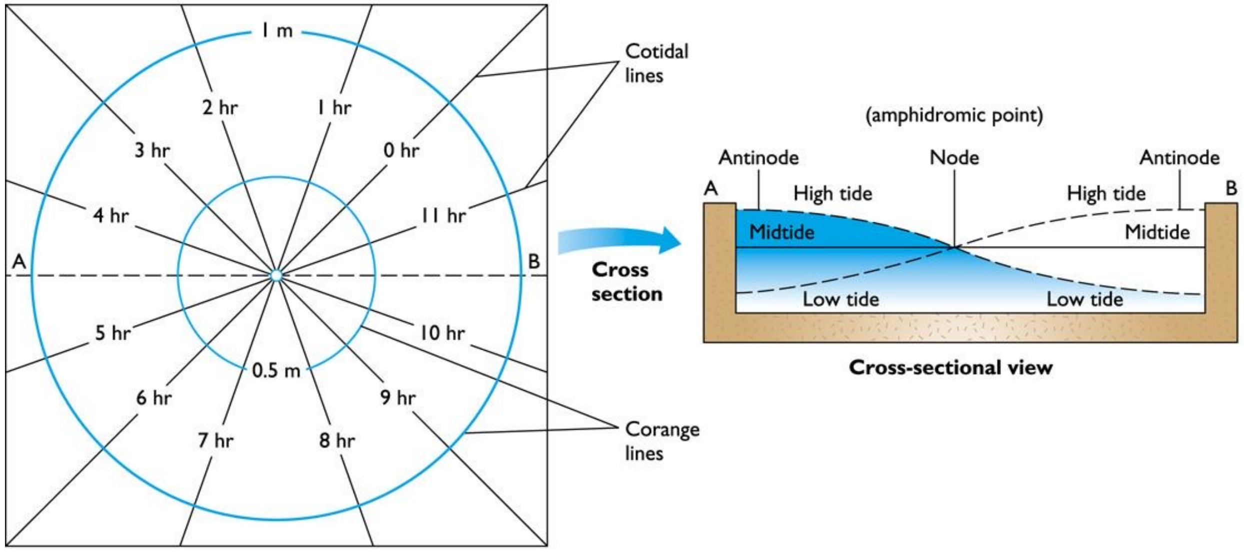
- Semi-Diurnal
- Diurnal
- Long Period
- Short period (not shown) – shallow water tides

Dynamic Tide Theory

- In reality, when the estimation of equilibrium tides does not provide the same result to what is actually seen in the ocean.
- This is because the worlds ocean has complex non-uniform bathymetry AND complex coastlines that impact the tides.
- Also, Coriolis force plays a role in causing the tidal wave to rotate in the horizontal direction for the full basin. (to the right – NH, to the left SH)
- This rotation is around a point, causing the formation of an amphidromic point (a point of minimum / no tides).



(a) IDEALIZED ROTARY TIDAL MOTION

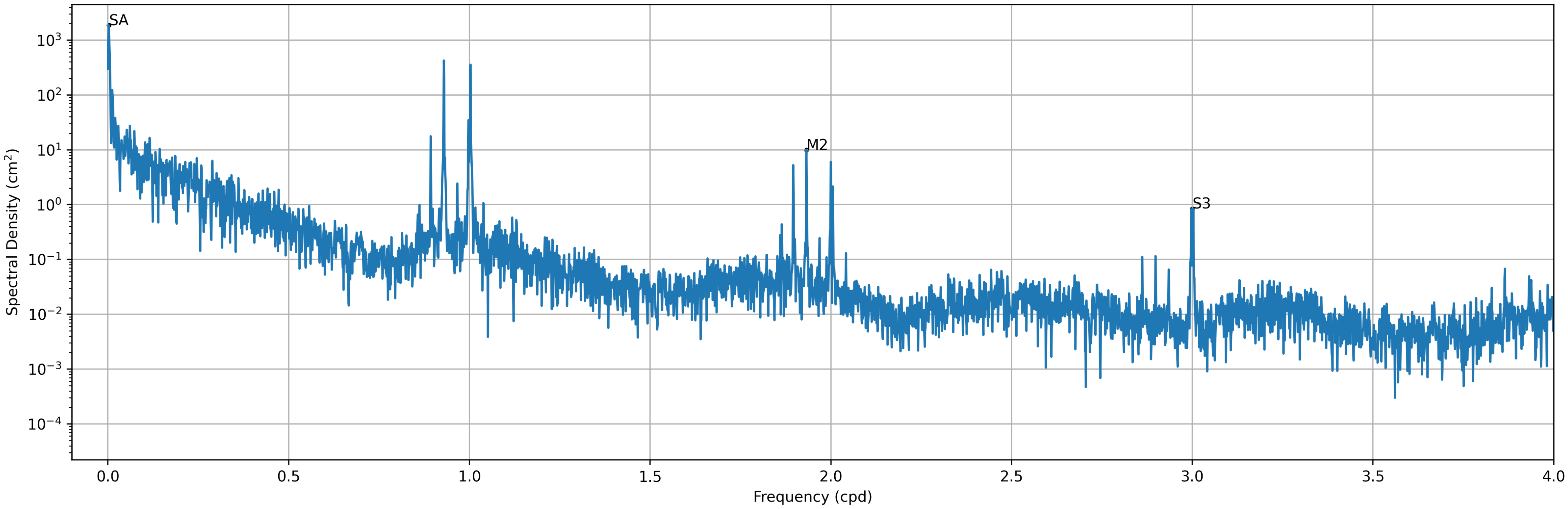


(b) AMPHIDROMIC SYSTEM

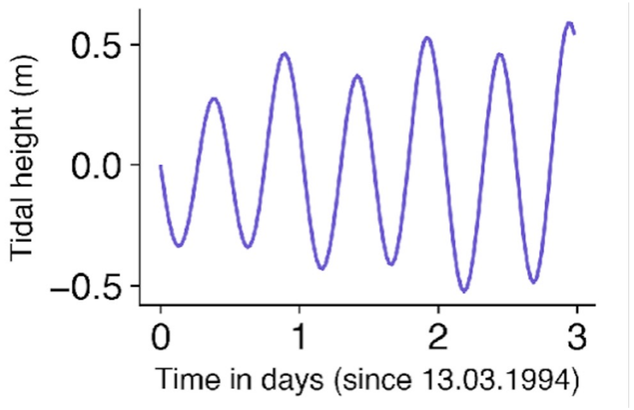
Tidal Constituents

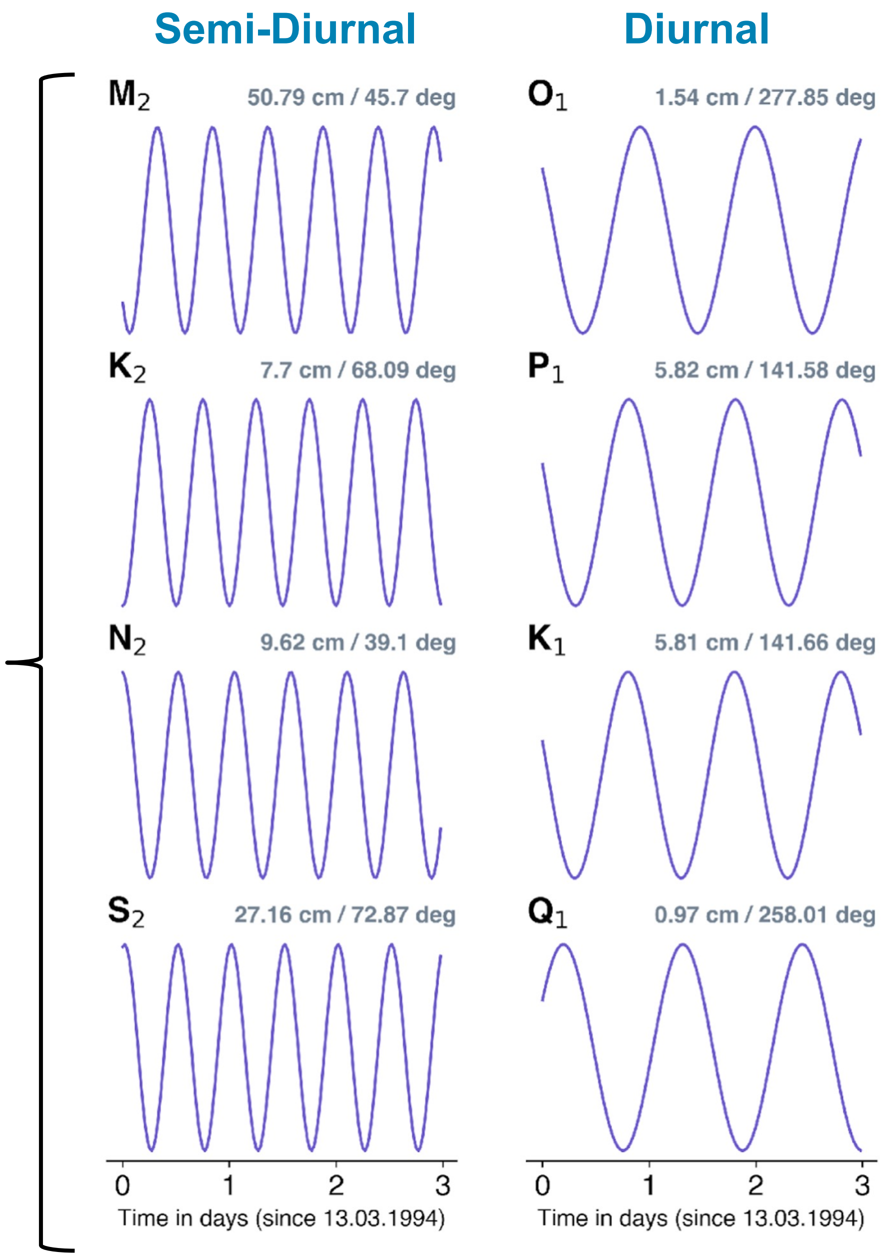
- The decomposition of the solar and lunar tides into a series of simple harmonic constituents.
- Spectral Analysis of Cuxhaven's (Germany) tide gauge

➢ SA – Solar annual tide
➢ M2 – Principle semi-diurnal lunar tide (**12.4 hr period**)
➢ S2 – Principle semi-diurnal solar tide (**12 hr period**)



Tide theory

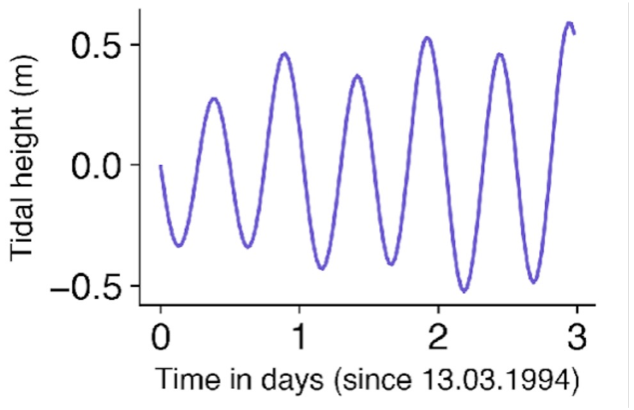




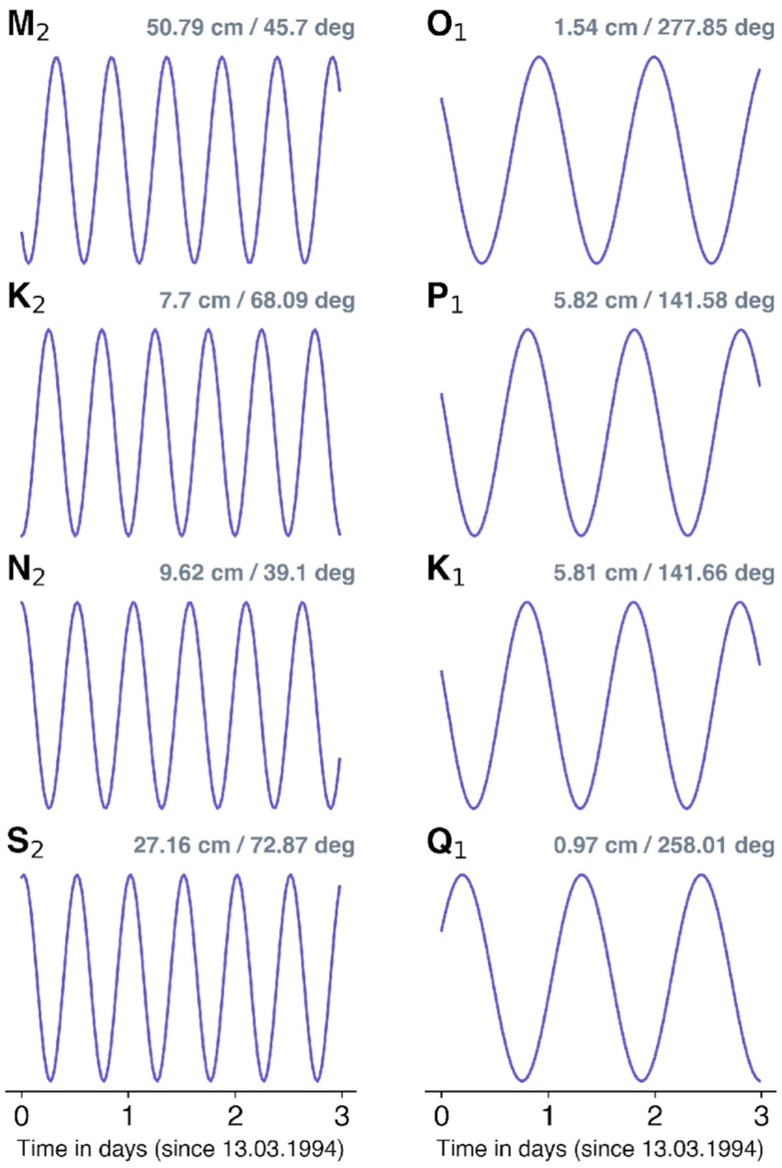
Harmonic Analysis:

$$H(t) = \sum_{i=1}^{i=N} A_i \cos(\omega_i t - \phi_i)$$

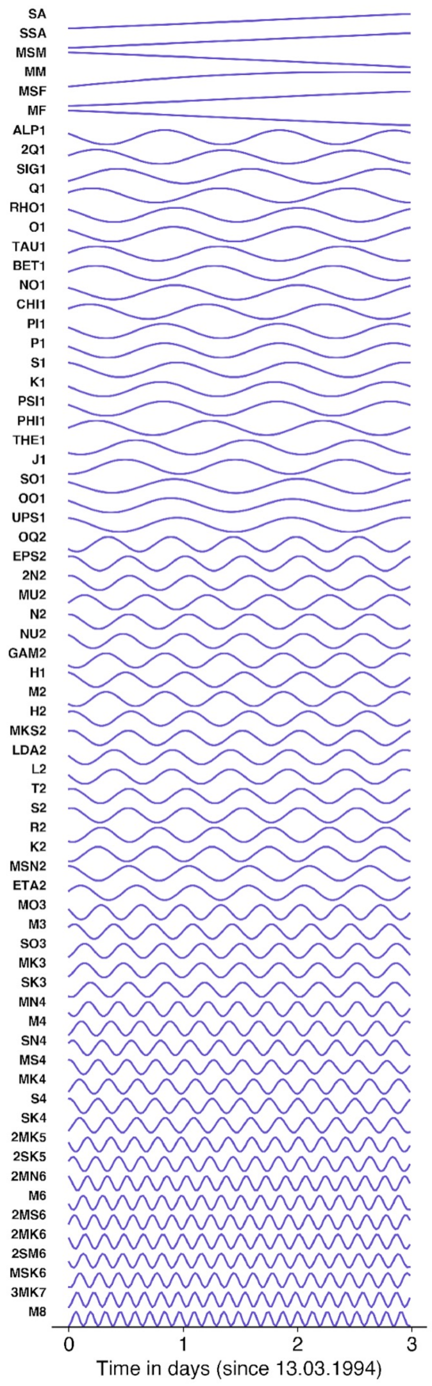
Tide theory



Tidal height (m)
0.5
0.0
−0.5
0
1
2
3
Time in days (since 13.03.1994)

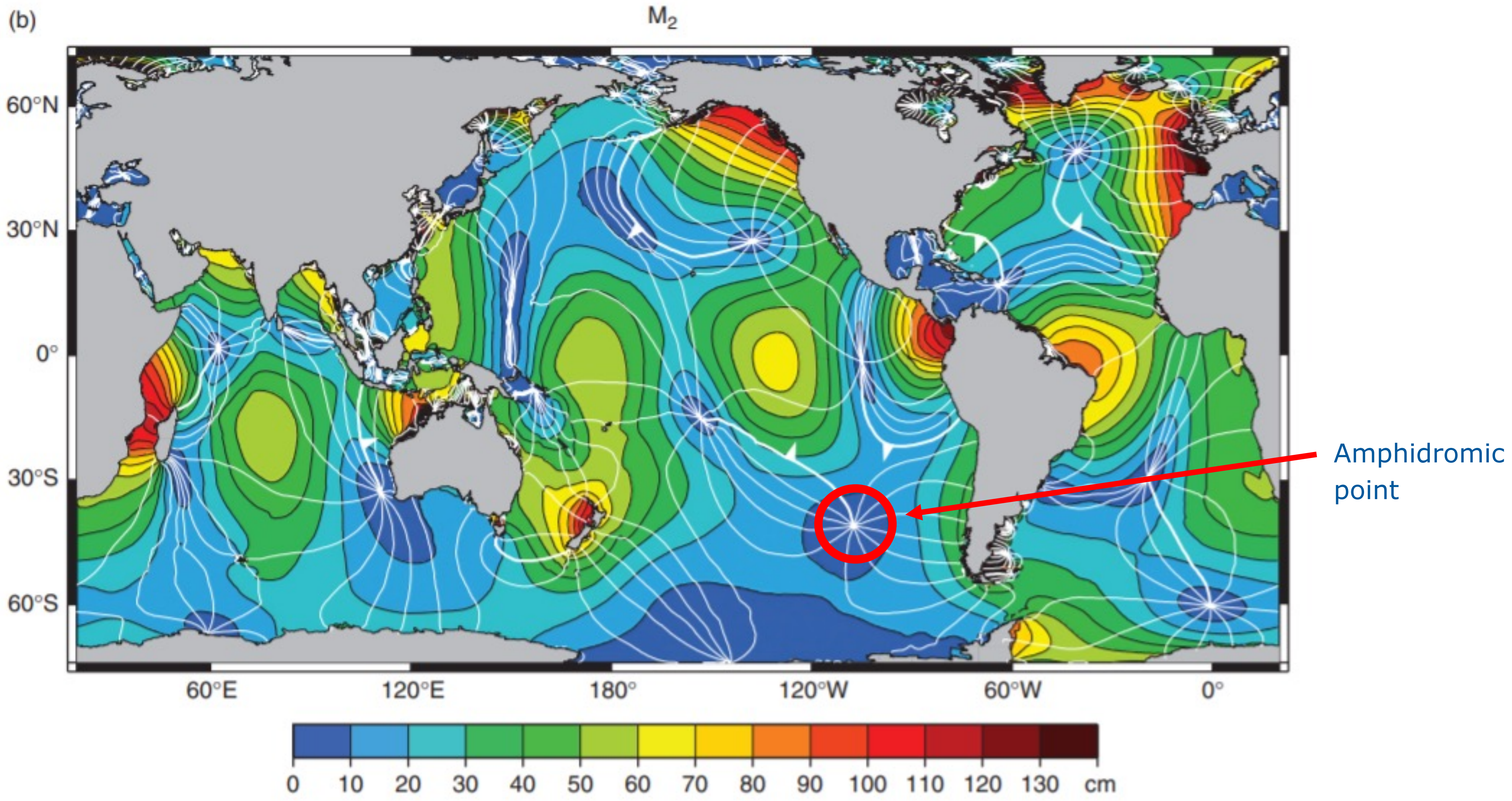


M2
50.79 cm / 45.7 deg
O1
1.54 cm / 277.85 deg
K2
7.7 cm / 68.09 deg
P1
5.82 cm / 141.58 deg
N2
9.62 cm / 39.1 deg
K1
5.81 cm / 141.66 deg
S2
27.16 cm / 72.87 deg
Q1
0.97 cm / 258.01 deg
0
1
2
3
Time in days (since 13.03.1994)
0
1
2
3
Time in days (since 13.03.1994)

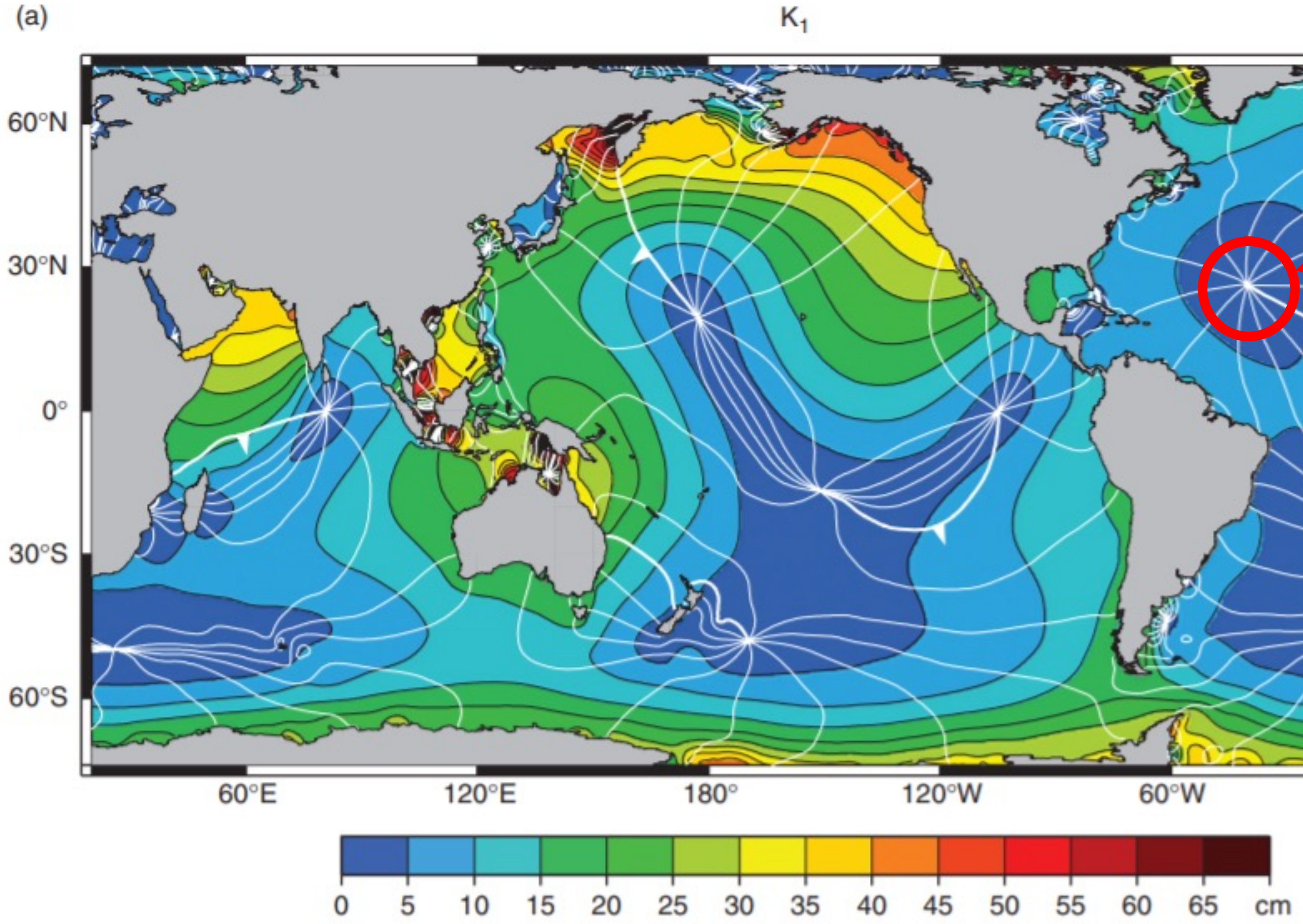


SA
SSA
MSM
MM
MSF
MF
ALP1
2Q1
SIG1
Q1
RHO1
O1
TAU1
BET1
NO1
CHI1
PI1
P1
S1
K1
PSI1
PHI1
THE1
J1
SO1
OO1
UPS1
OQ2
EPS2
2N2
MU2
N2
NU2
GAM2
H1
M2
H2
MKS2
LDA2
L2
T2
S2
R2
K2
MSN2
ETA2
MO3
M3
SO3
MK3
SK3
MN4
M4
SN4
MS4
MK4
S4
SK4
2MK5
2SK5
2MN6
M6
2MS6
2MK6
2SM6
MSK6
3MK7
M8
0
1
2
3
Time in days (since 13.03.1994)

The structure and magnitude of these constituents vary

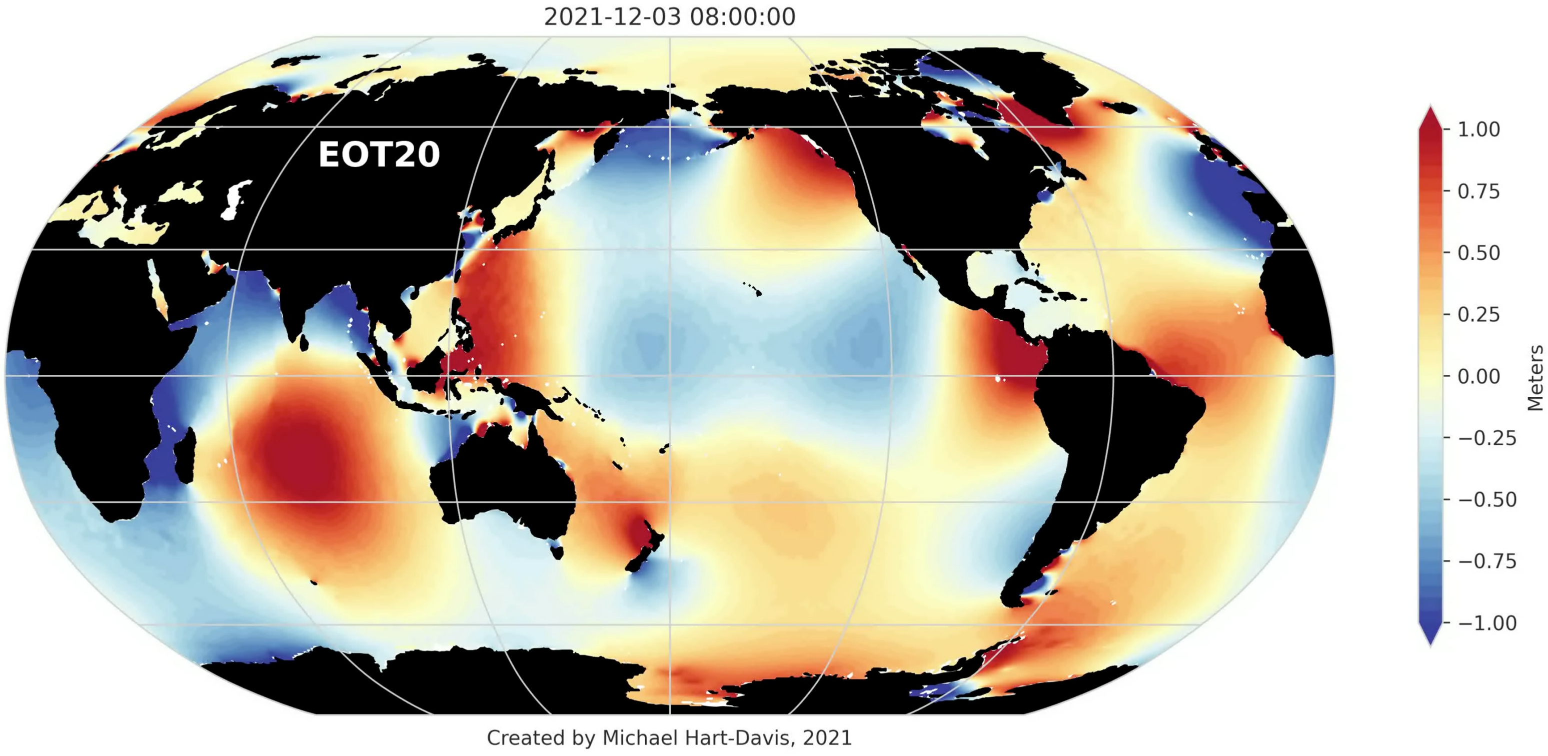


The structure and magnitude of these constituents vary



Amphidromic point

Ocean tides vary throughout the ocean

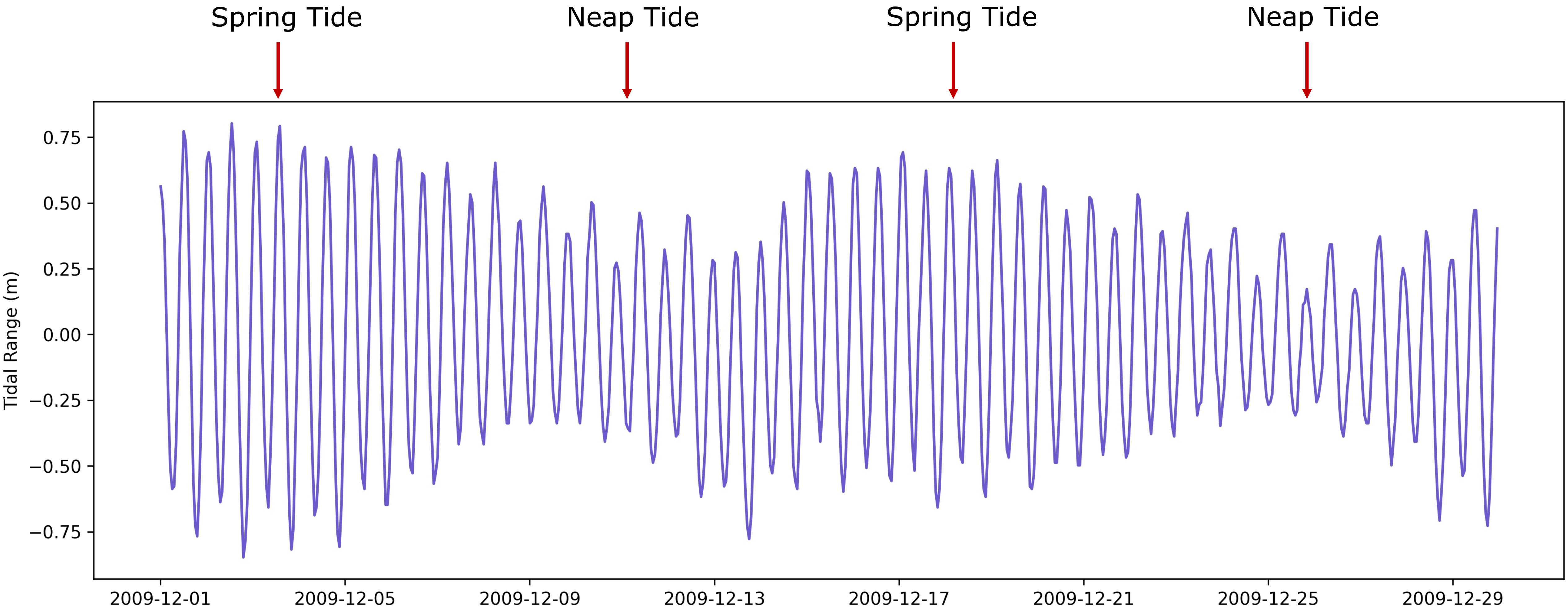


Ocean Tidal Heights Developed by DGFI-TUM, the Empirical Ocean Tide Model (EOT20).

- The tides in the open ocean (~1 meters) are considerably smaller than in the coastal region (sometimes exceeding 15 meters).
- This is due to the tidal wave propagating into the shallower regions, causing a rise in the sea level (higher tides) compared to the surrounding deeper oceans.
- The gradient of the topography contributes significantly, with gentler slopes resulting in a higher tidal range.
- The coastline shape also influences the tides, with narrower regions (such as bays) also amplifying the tidal range. We can see this in the English Channel.
- Topography type also is important (rocky shore, kelp forests etc)

The Changes of Tides over Time

Spring Tide: when the Moon and Sun align along the Earth-Moon line.
Neap Tide: when the Moon and Sun ‘counter-act’ each other by being the Sun being at 90 degrees to the Earth-Moon line

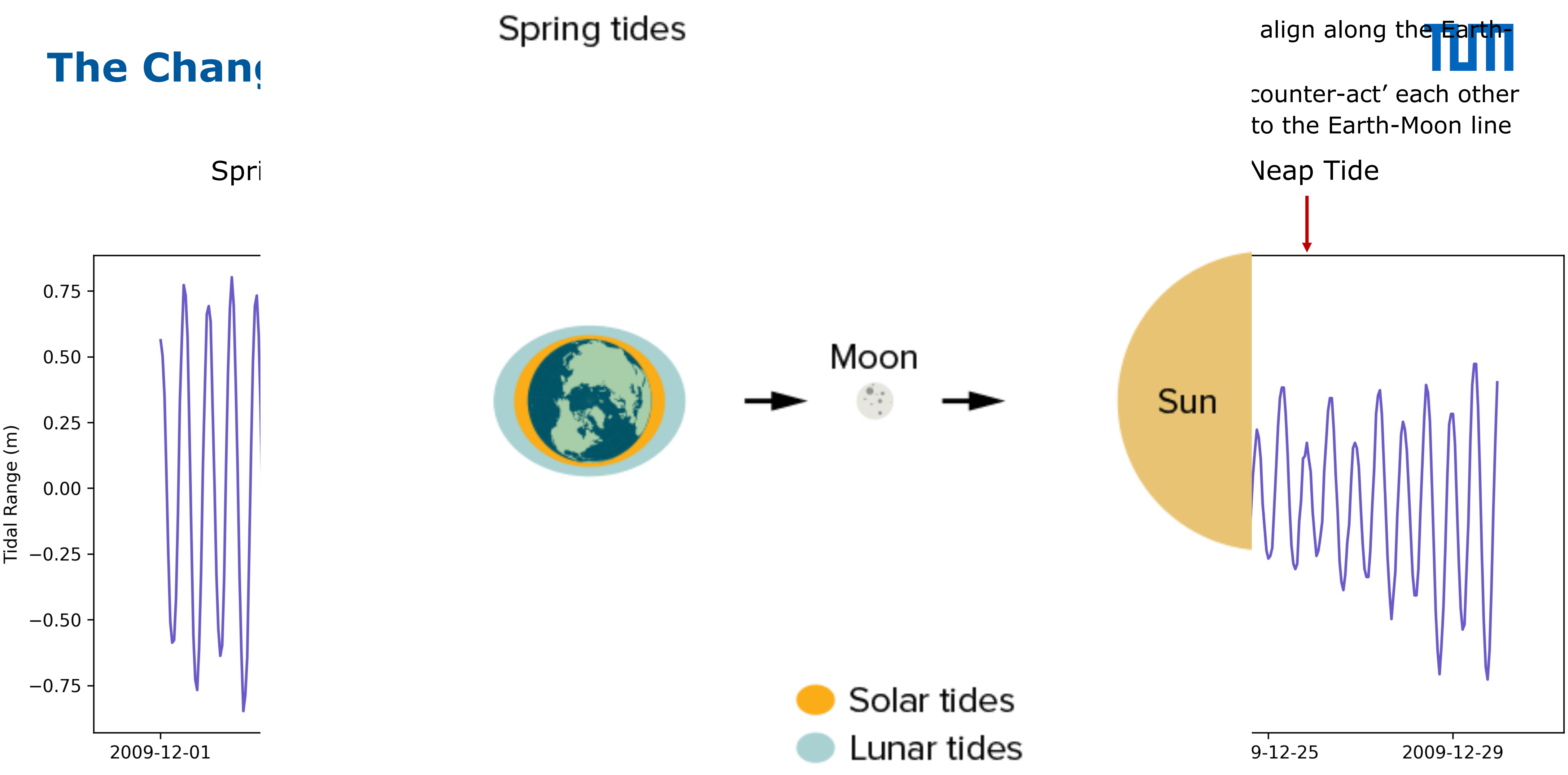


The Chan

align along the Earth-

counter-act' each other

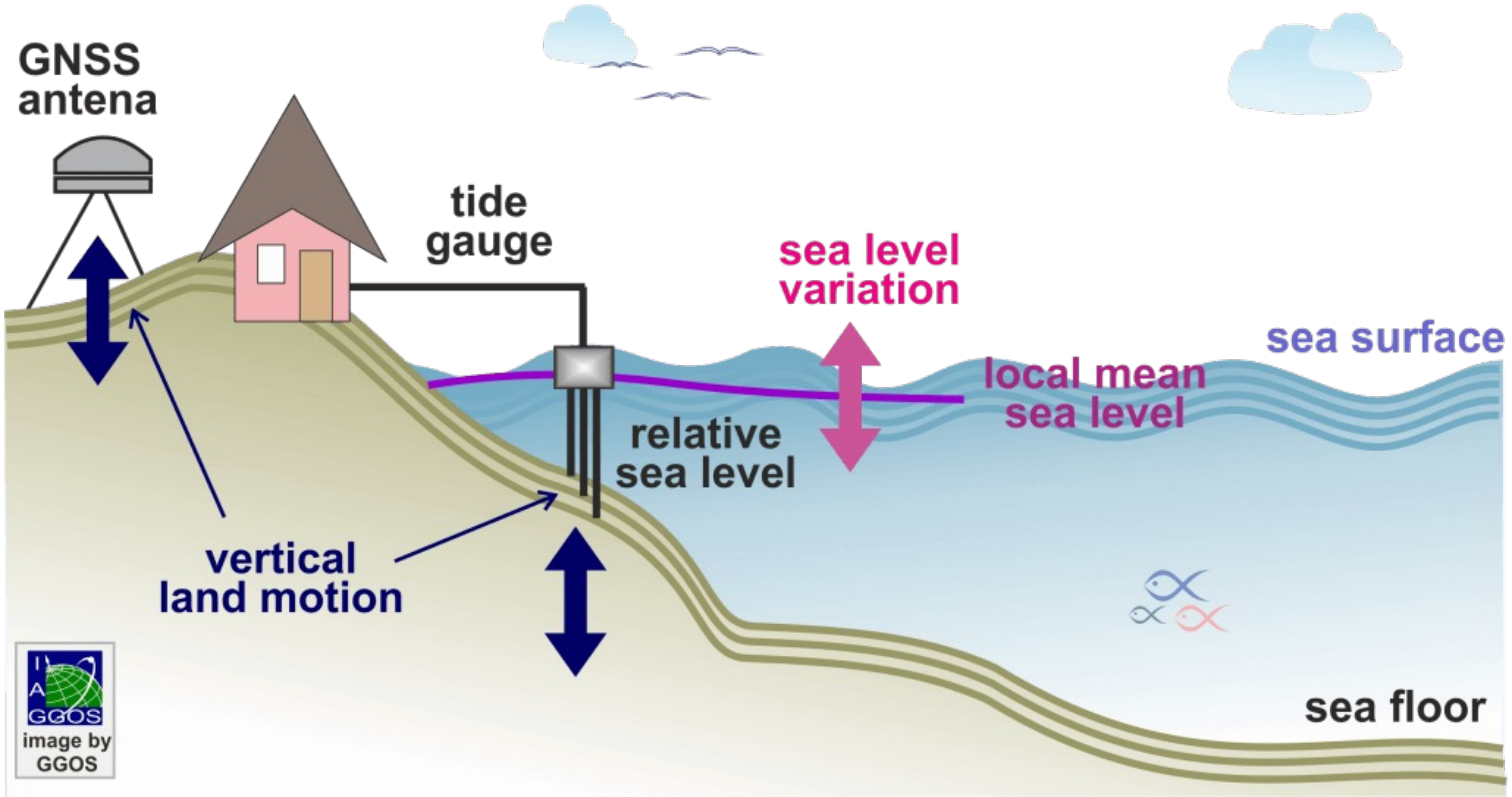
to the Earth-Moon line



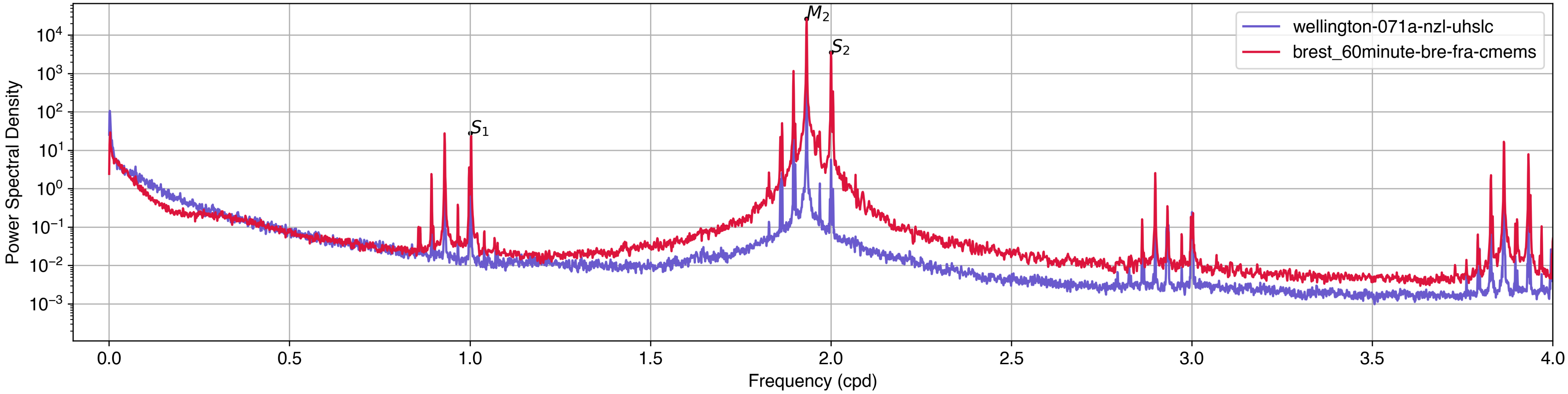
Ocean Tide Observations and Ocean Tide Models

How do we measure / estimate tides?

- Tide Gauges
- Ocean Bottom Pressure Sensors
- Satellite Altimetry
- Ocean Tide Models
- Moorings



Tides from Tide Gauges

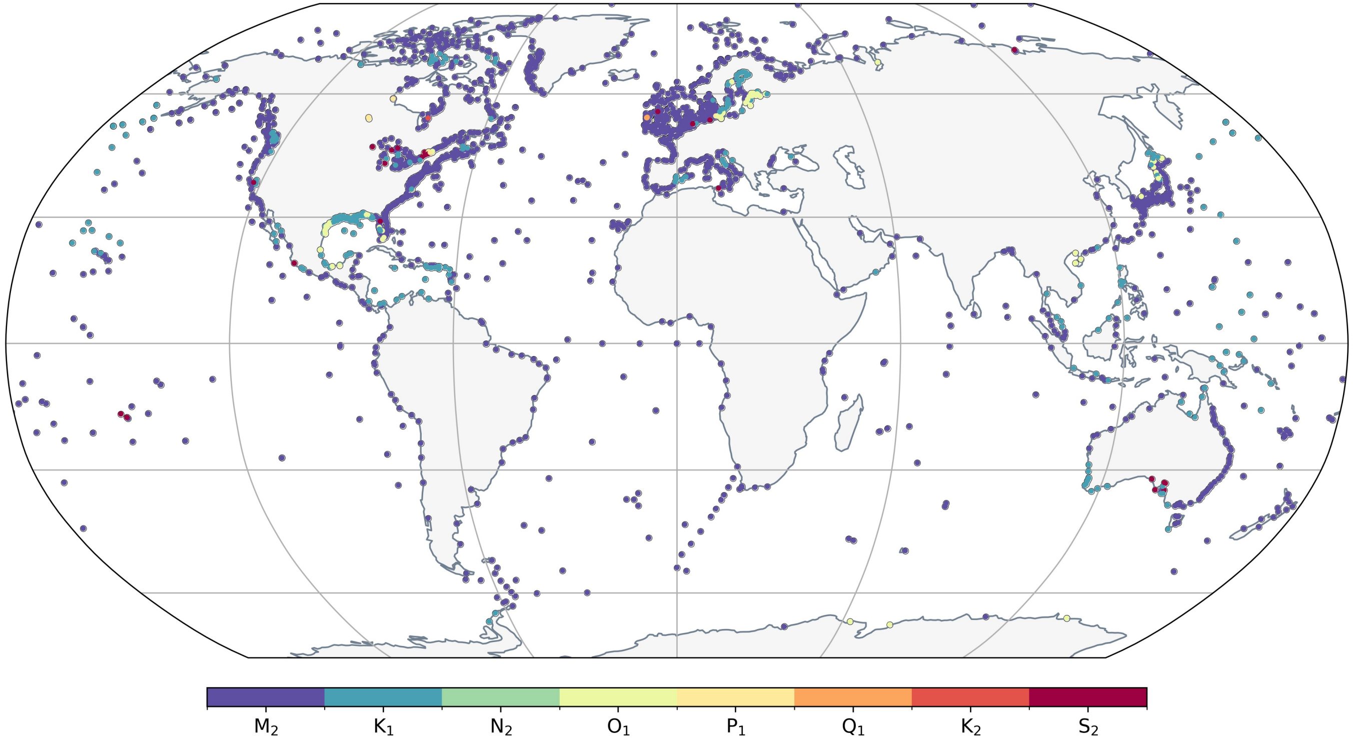


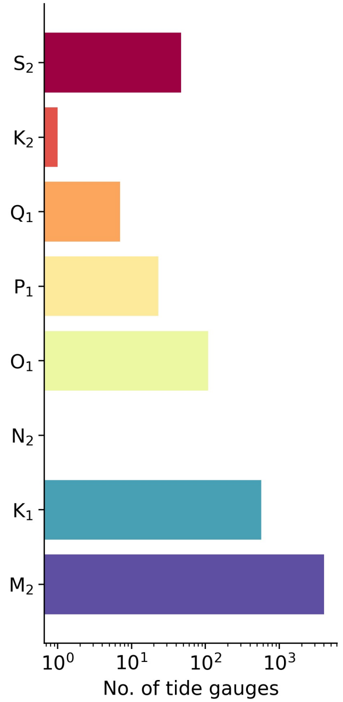
- As their names suggest, tide gauges are extremely valuable for tidal estimations.
- We can nicely see within the Power Spectral Density, that the known frequencies of tides show up and are the dominant frequency processes.
- Tide gauges have formed the fundamental basis of tide understanding and modelling for centuries, with early models using tide gauges to produce tidal estimations.
- These gauges are also used in model validations as well as assimilation

Difficulty in accurately understanding ocean tides

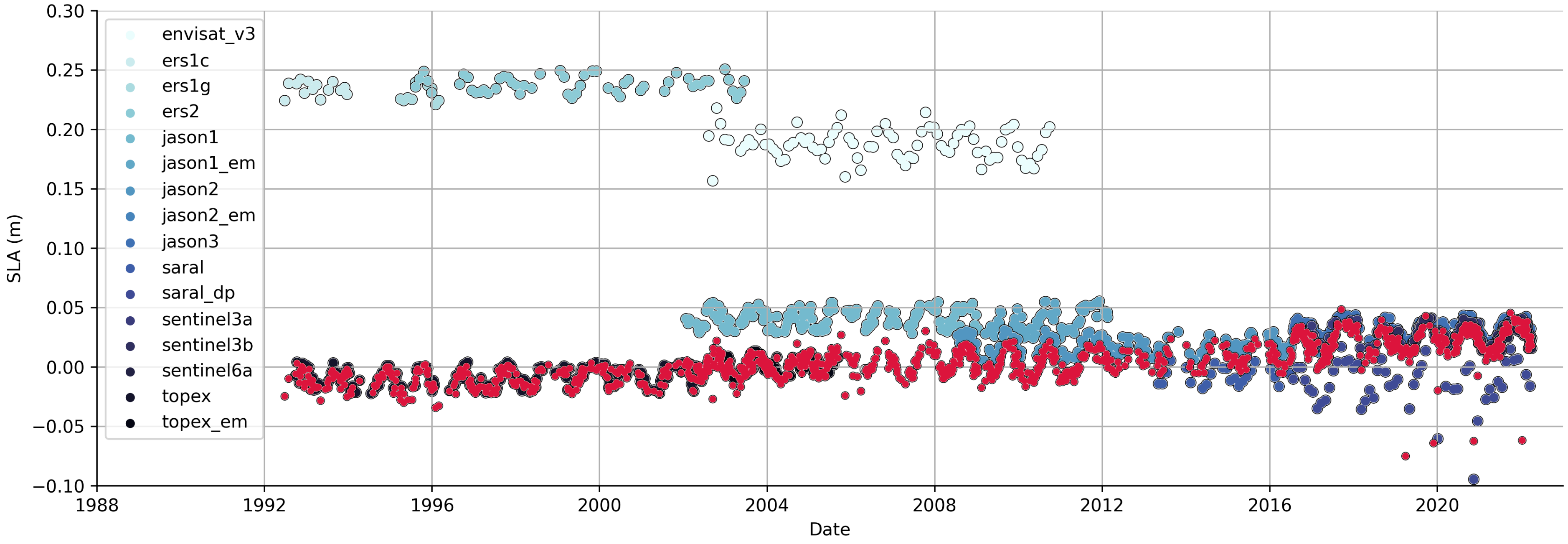
Problems in estimating ocean tides frequently occur in the coastal region. This is caused by:

- The lack of properly distributed observations.
- Poorly understand bathymetry.
- Radar returns of satellite altimetry more strongly affected in the coastal region.
- Requires very high computational load to more accurately estimate all the tidal constituents needed to resolve the full ocean tide.
- Sea Ice



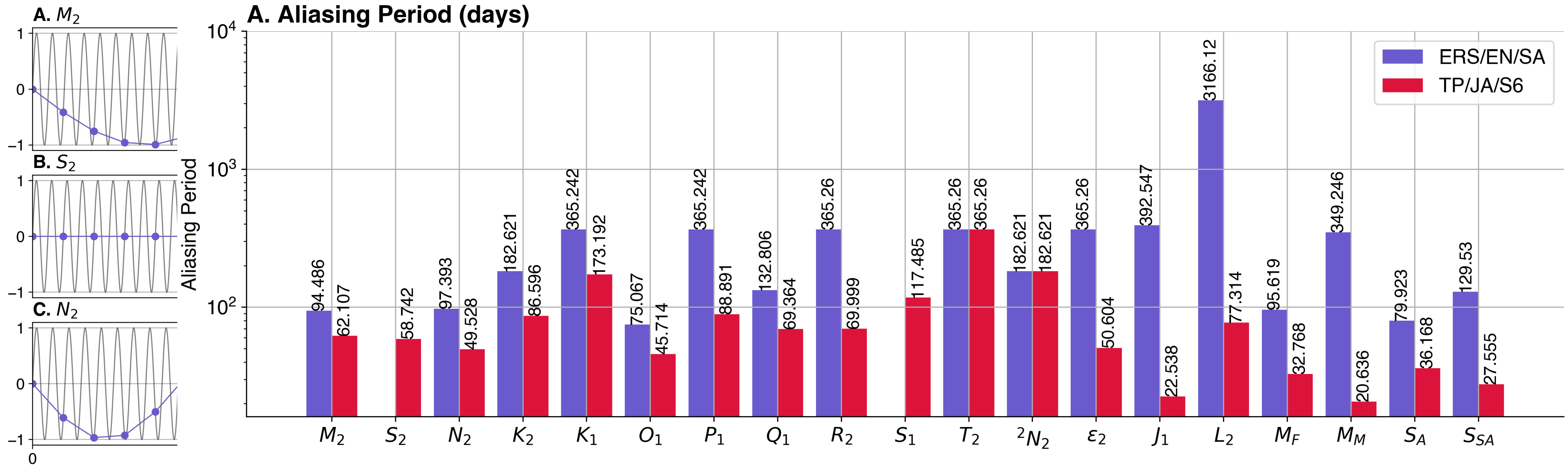


Tides FROM Satellite Altimetry



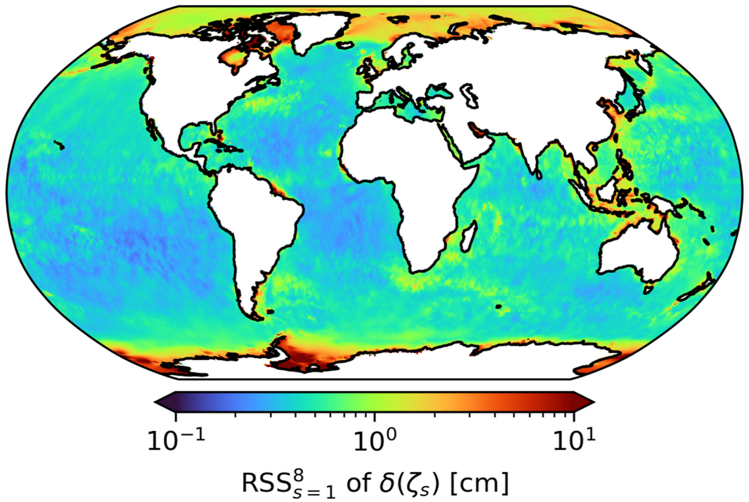
- Over thirty years of data from multiple satellite altimeters is available.
- Several of these missions fly of different orbits, but using appropriate gridding strategies we can bin them together to get time-series at points across the globe.
- Using tidal analysis techniques, such as harmonic analysis or residual analysis, we can derive tidal from these time-series.
- Using these time-series, we can therefore make estimates of the tidal height which is used as a tidal correction.

Tides FROM Satellite Altimetry: Aliasing Issues



- Tidal constituents occur at known frequencies. But due to the sampling frequency of modern altimeters, these tides are aliased onto much longer periods.
- This means that longer time-series of data is required to estimate these tides.
- For most tides, the altimetry datasets are fine for resolving these tides (except for extremely long period tides OR tides whose frequencies get aliased to decades).
- Problems DO occur in sun-synchronous orbit satellites which make the aliasing periods infinite for solar tides.

State of the Art of Tide Models



Hauk et al., 2023; RSS of the five models used in the NEROGRAV project to derive the OTVCMs.

Latest global tide models developed in recent years:

- GOT5.6 (Ray 2025)
- DTU22 (Andersen, Rose, Hart-Davis, 2022)
- ***EOT20*** (Hart-Davis et al 2021)
- FES2022 (Lyard et al 2025, in prep)
- TPXOv10 (Egbert and Erofeeva, 2002)
- ***TiME22*** (Sulzbach et al 2022)

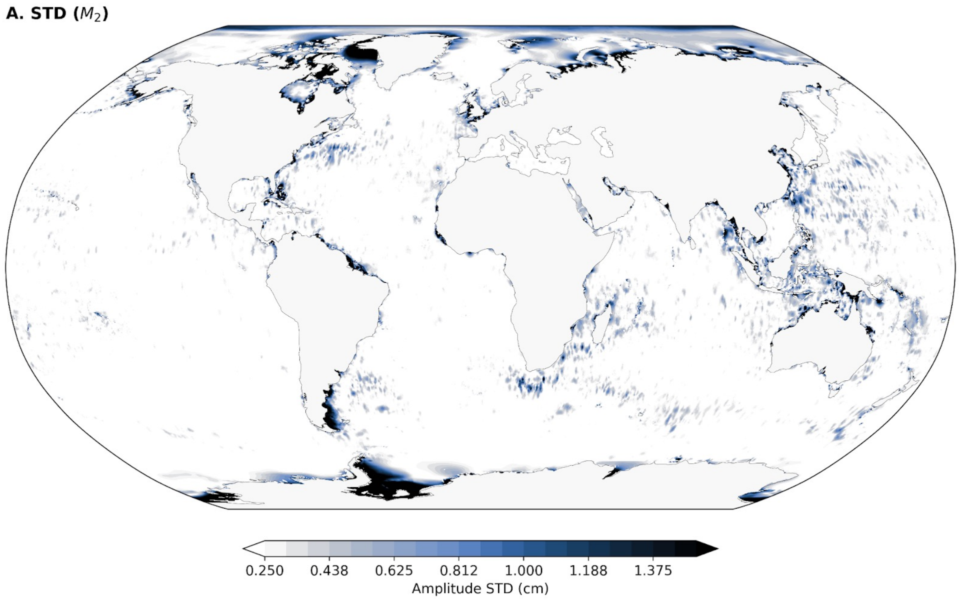
Global models are typically characterized into three types:

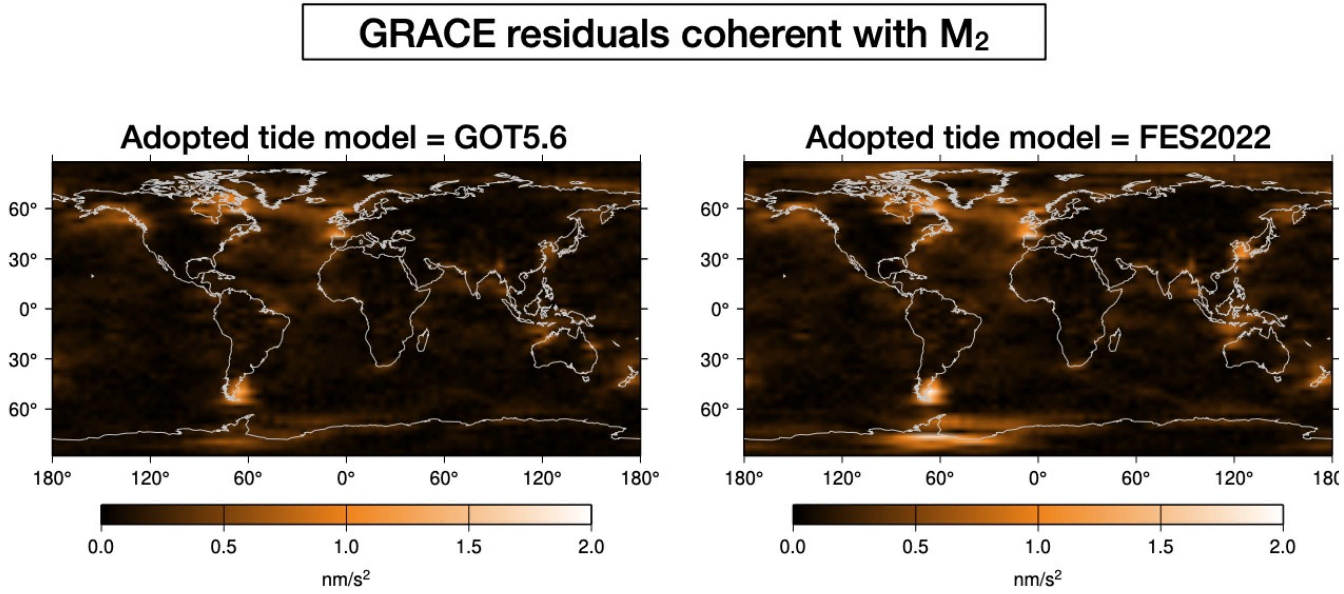
1. Semi-empirical
2. Data assimilative
3. Numerical models

The choice of these model types has their own strengths and weaknesses:

- Typically, the largest tidal signals or constituents are better resolved by the data driven approaches thanks to satellite altimetry.
- Numerical models can derive a lot more tidal constituents at sufficient accuracy, which is crucial for resolving the full tidal signal which can be hundreds of tidal constituents.

Even state of the art models have challenges





Standard deviation of the M2 tidal amplitude from five most recent data driven models (FES2022, EOT20, TPXO10, GOT5, DTU22).

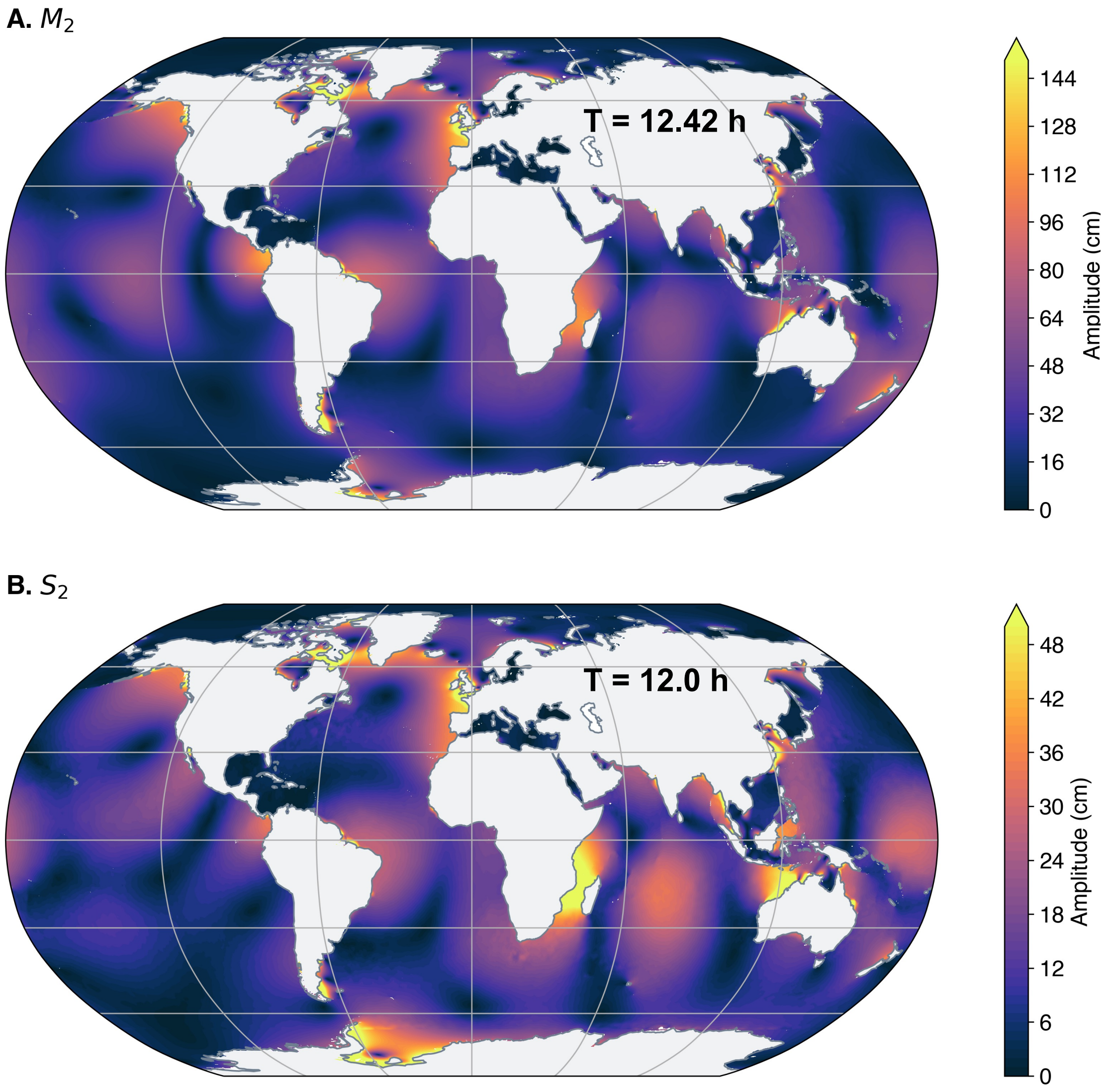
- Biggest variations and errors to tide gauges are seen in the coastal and polar regions.

GRACE residuals with updated tide models (GOT5.6 and FES2022), provided by Richard Ray[1]. Simulations run by Chaoyang Zhang.

- Residuals are largest in coastal and polar regions, likely a result of tide model errors, complexity of tides in the regions and the limited number of tidal constituents in these regions.

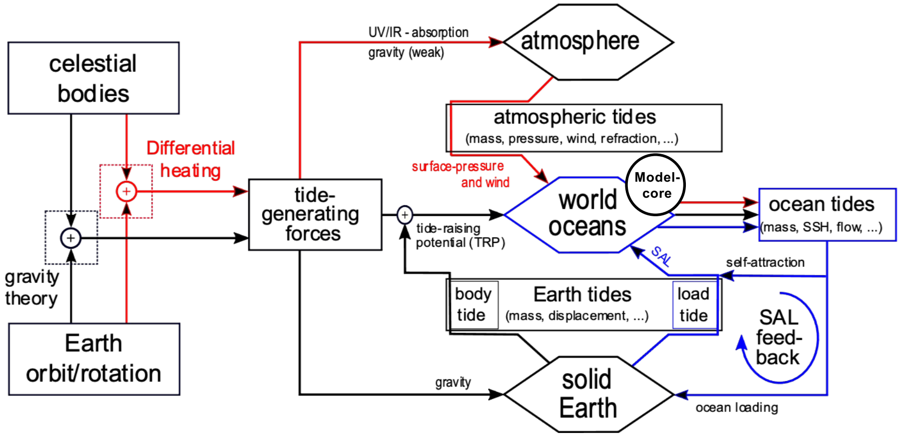
EOT20

- Empirical Ocean Tide (EOT) model developed for several decades at DGFI-TUM.
- The model is based on a residual harmonic analysis of along-track satellite altimetry.
- The latest model, EOT20, focuses on coastal performance and is derived from nearly 30 years of multi-mission satellite altimetry.
- EOT20 contains 17 tidal and validation with in-situ measurements demonstrated the highest level of accuracy of the model within the coastal region.



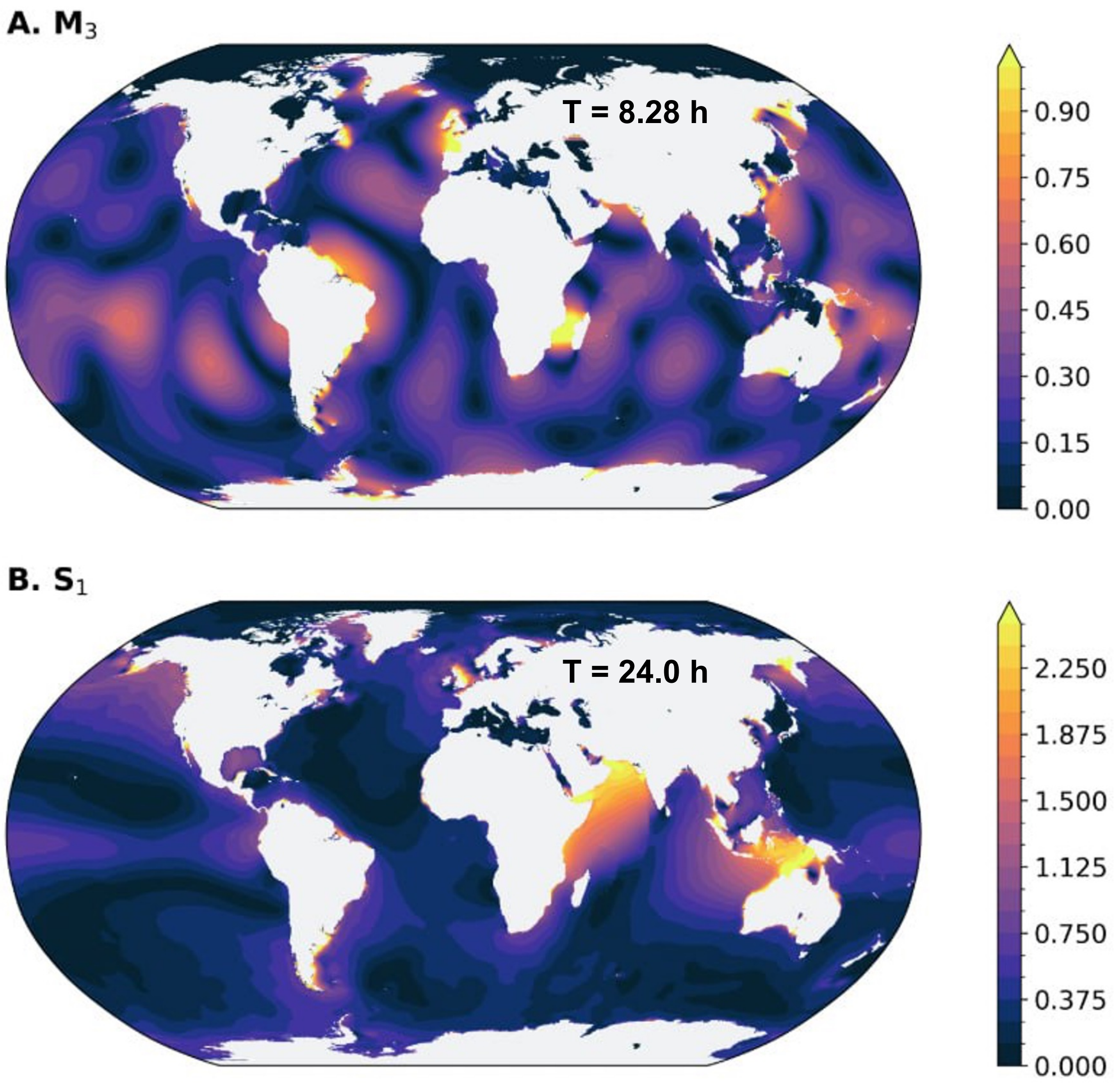
TiME22

- Purely-numerical ocean tide model developed at GFZ.
  - Applies no empirical observations
  - Numerical integration of the shallow water equations on a rotated-pole grid.



- Excitation of ocean tides by tide generating-forces and interaction with Earth and atmospheric tides.

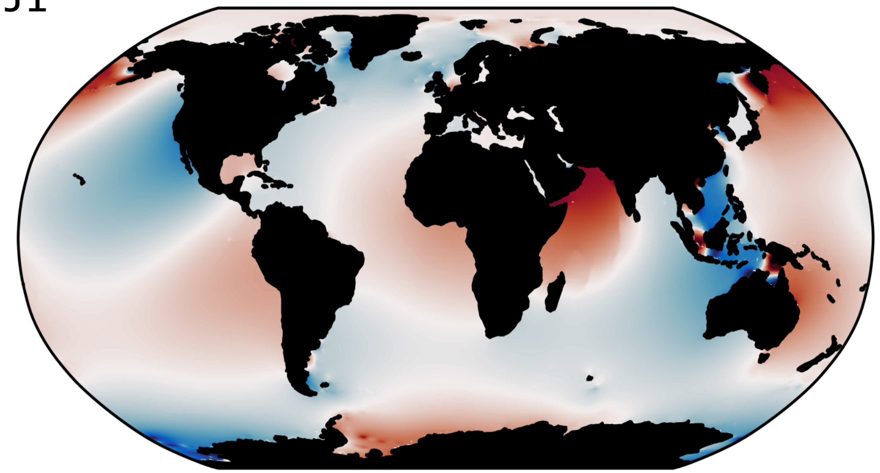
- Prediction of a large number of partial tides (50+) with a focus on minor tidal constituents with small amplitudes.



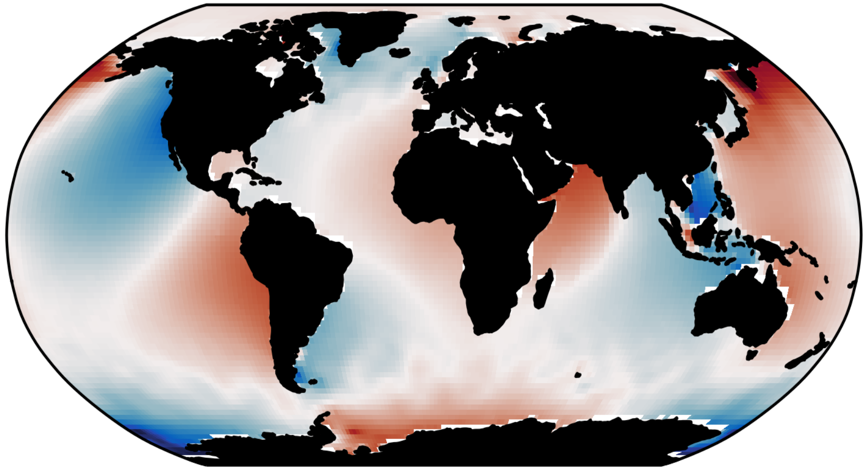
Admittances

- Ocean tide models provides only information of the largest tides (main waves)
- Smaller waves (minor tides) can be inferred by interpolation from main waves
- Assumption: relation of the tidal height with respect to the amplitude of the corresponding tide-generating potential (TGP) is a smoothed function of the frequency
- Most common: assumption of linear variation of tidal admittance between closely spaced tidal frequencies
- For example: O1/K1 => J1

J1 Constituent (Acos) - Admittance FES



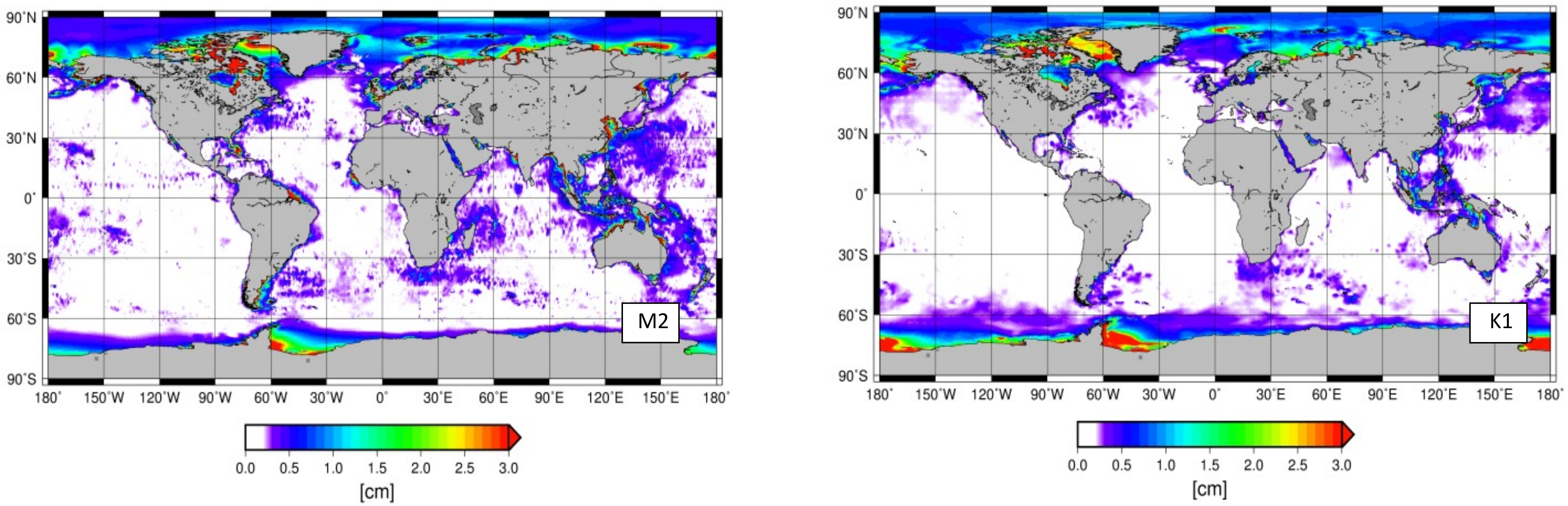
J1 Constituent (Acos) - Original FES



Model uncertainties

Accuracies of ocean tide models

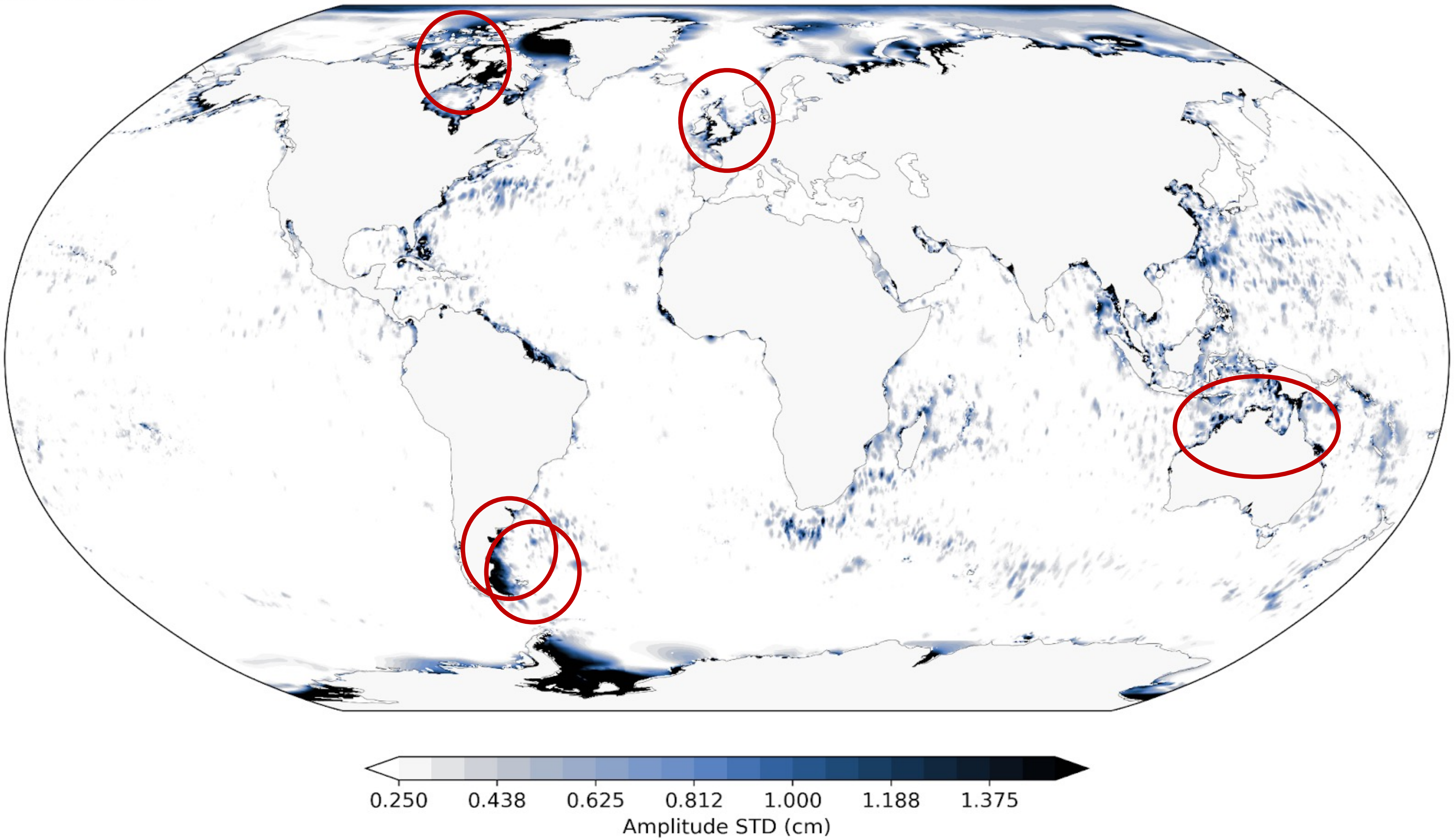
- Uncertainties are not provided together with the models.
- Information can be inferred by comparing different models to each other.



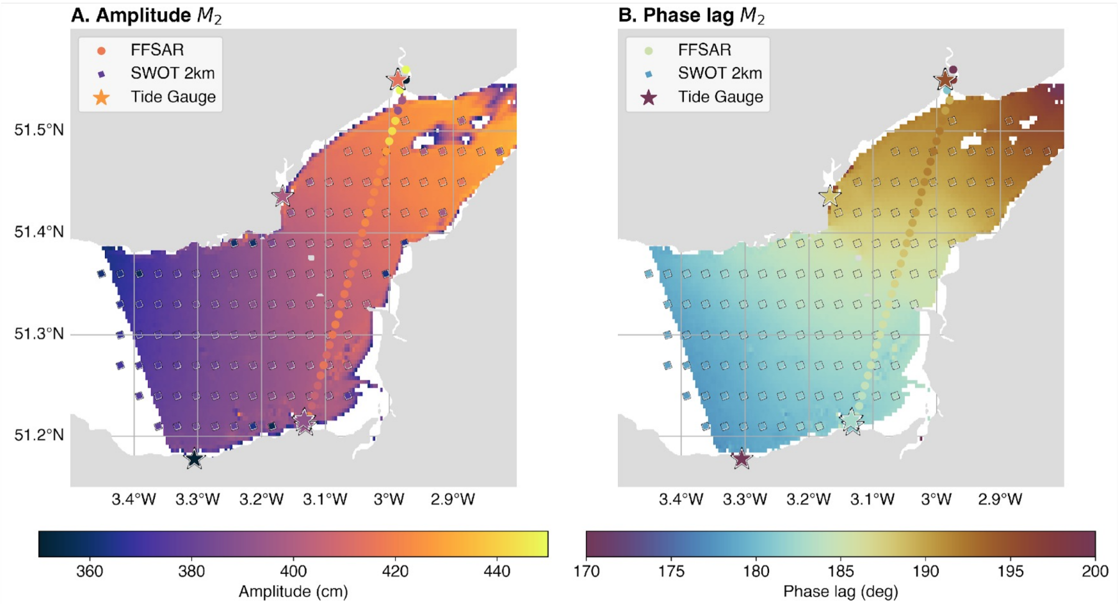
Standard deviations (SD) between five different ocean tides models following Stammer et al. (2014)

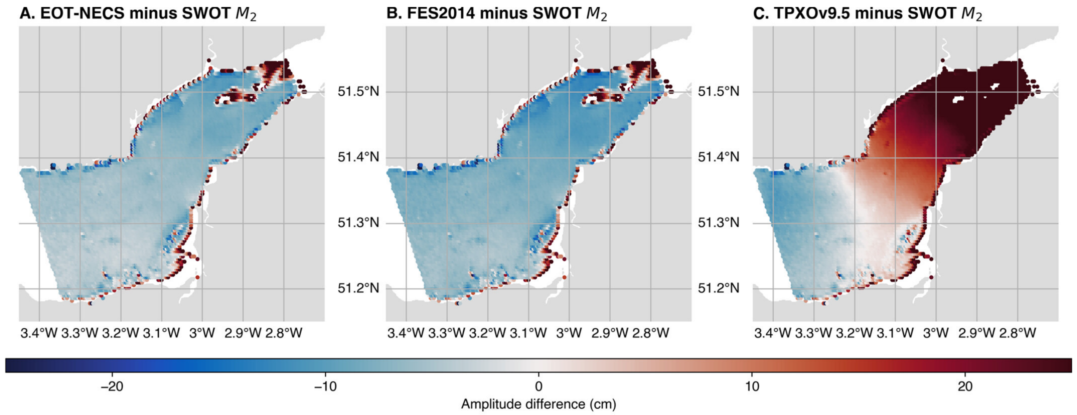
Critical regions

A. STD (M_2)



Latest Research: NEROGRAV tries to improve the coasts!





- The newly launched SWOT satellite, allows for measurements of sea level at 50-meter resolution.
- For the first time, we used these data to derive tidal estimates in extremely nonlinear and complex coastal regions.
- With respect to tide gauges from TICON-3 (Hart-Davis et al 2021) we found mean differences of 2.58 cm and 2.72 degrees for the amplitude and phase lag, respectively.
- In this region, regional (EOT-NECS) and global models showed errors exceeding tens of centimeters and degrees.

For more information: Hart-Davis et al (2024)

Latest Research: NEROGRAV tries to improve the Polar oceans!

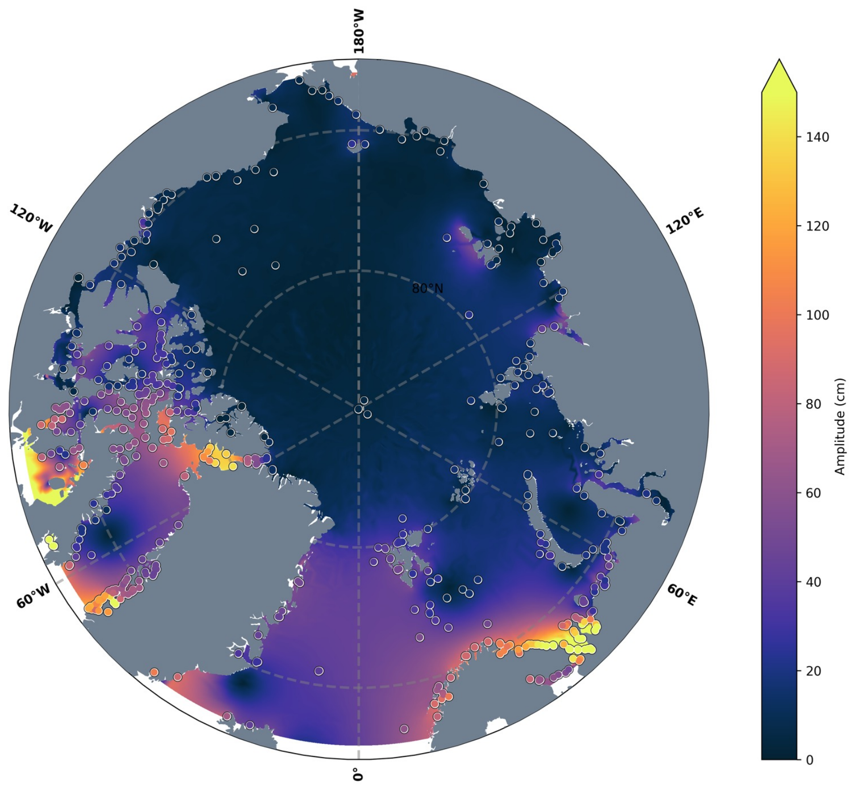
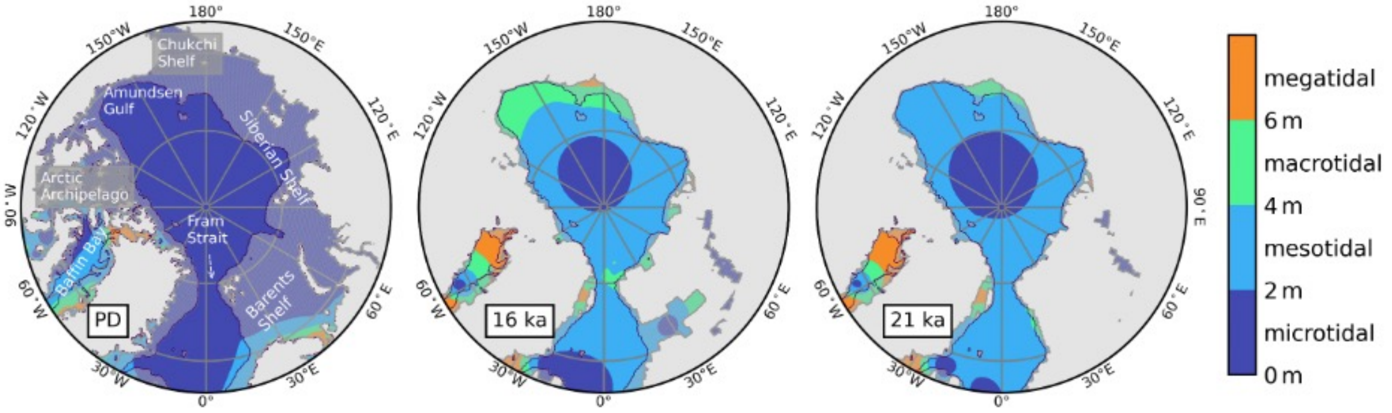


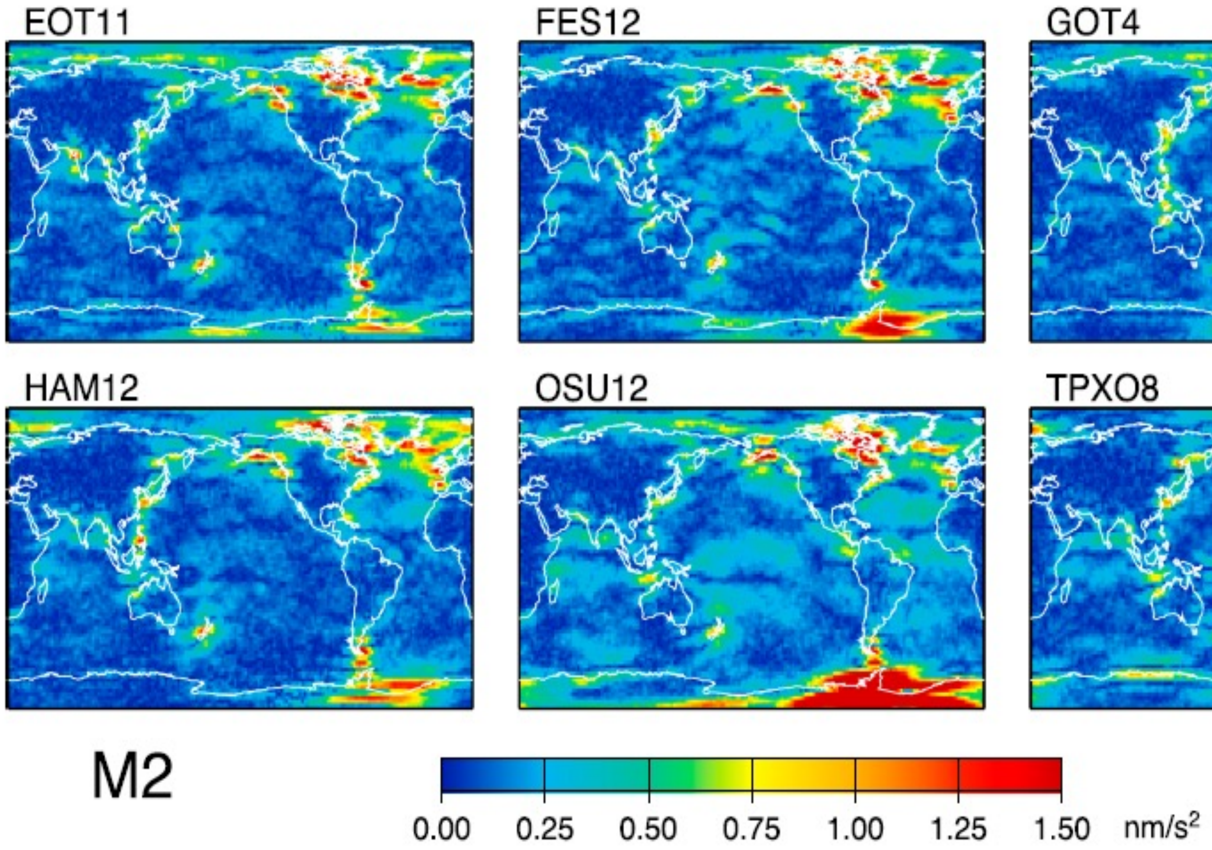
Fig. Amplitude of the M2 tide in the Arctic Ocean derived from EOT.

- High frequency processing done at DGFI-TUM aimed at improving the quality of sea level anomaly (SLA) measurements and the quantity.
- The model now contains data from 2010 – 2024, and is of nine altimetry datasets, and produces a significant reduction in constituent error.
- Undestanding how tides have varied throughout history (***Sulzbach et al 2025***):



Remaining errors in ocean tide models and its impact on GRACE

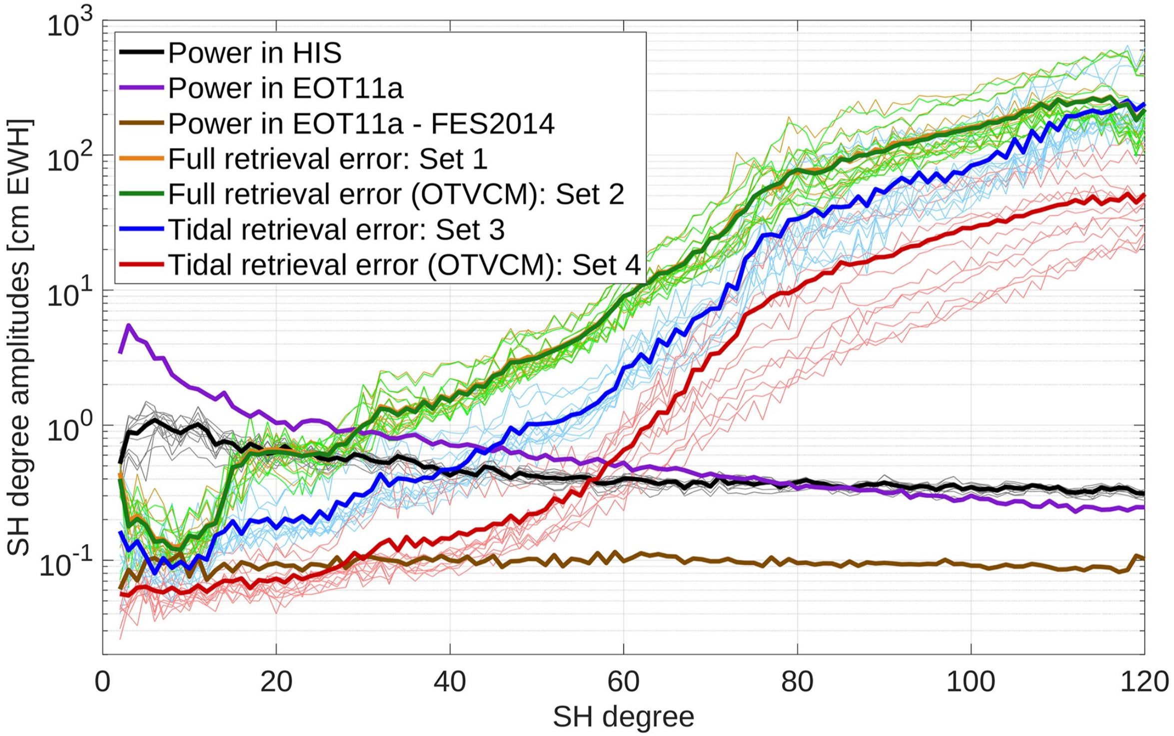
Remaining uncertainties in GRACE data



Amplitudes at the M2 tidal frequency in 7 years of GRACE range acceleration residuals, based on six different tidal models.
Locations showing significant amplitudes suggest errors in the corresponding tide model. [Stammer et al., 2014]

Latest Research: NEROGRAV tries to account for Uncertainties

- As no state-of-the-art tide models provide uncertainty data, a large effort to derive uncertainty estimations was done by producing variance-covariance matrices from 5 global tide models.
- These error information were then applied to GRACE data and evaluated to see whether the error statistics had an impact on water height (EWH) estimations and the tidal retrieval errors.
- Work was published in **Hauk et al (2023):** https://doi.org/10.1029/2023EA003098



The averaged ocean wRMS based on tidal retrieval with and without OTVCM (cf. Figure 4e) is 2.95 cm EWH and 0.75 cm EWH, respectively, resulting in a **reduction of tidal aliasing errors of about 75%.**

Summary / Take-home message

Ocean Tides

- Knowledge of ocean tides is important – not only for most obvious applications (such as coastal protection)
- A variety of global hydrodynamic and empirical ocean tide models is available. Usually, no uncertainties are provided as part of the models
- The models' uncertainty (models' spread) is worst in coastal and polar areas
- In NEROGRAV we are working on the improvement of ocean tide models (TiME and EOT) and on the determination of reliable model uncertainites
- Realistic model uncertainties are expected to help in gravity modelling

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